COMPREHENSIVE REVISION QUESTIONS IN A' LEVEL STATISTICS, PROBABILITIES AND NUMERICAL METHODS.

STATISTICS.

1. The information below shows the test marks of 11 students.

   52,61,78,49,47,79,54,58,62,73,72

   Find;

   a) Median

   b) Mean

   c) Lower quartile

   d) Upper quartile

   e) Interquartile range

   f) Semi-interquartile range

   (Answer: a) 61 , b) 62.273, c) 52, d)73, e)21, f) 10.5

2. The information below shows the test marks of 11 students.

   5, 6, 7, 4, 4, 7, 5, 4, 5, 8, 6, 2, 7, 3, 7

   Find;

   i) Median

   ii) Mean

   iii) Lower quartile

   iv) Upper quartile

   v) Interquartile range

   vi) Semi-interquartile range

   (Answer: i) 6.875 , ii) 6.50 , iii) 6, iv) 3.609, v) 1.90, vi) 3

3. Given the information below showing the test marks of 8 students.

   8, 10,7,6,4,5,6,9

   Find:

   a) Mean

   b) Median

c) Mode

d) Variance

e) Standard deviation

f) Interquartile range

(Answer:a) 10.857 , ii) 12 , iii) 6, iv) 3.609, v) 1.90, vi) 3

4. Given the marks below of 7 students marked out of 15

   10, 12, 13,15,8,12,6

   Calculate:

   a) Mean

   b) Median

   c) Mode

   d) Variance

   e) Standard deviation

   f) Semi-interquartile range

5. For a particular set of observations $n = 20, \varepsilon x^2 = 16143, \varepsilon x = 563.$

   Find the values of the mean and the standard deviation.

6. For a given frequency distribution $\varepsilon(x - \bar{x})^2 = 182.3,\ \varepsilon x^2 = 1025, n = 30$

   Find the mean and standard deviation of the distribution. (ans; $\bar{x} = 5.297, \delta = 6.11$)

7. For a set of 9 numbers $\varepsilon(x - \bar{x})^2 = 60$ and $\varepsilon x^2 = 285$. Find the mean of the number.

8. The mean of 10 numbers is 8. If an eleventh number is now included in the results, the mean becomes 9. What is the value of the eleventh number?

9. The mean of the numbers 3, 6, 7, a, 14 is 8. Find the standard deviation of the set of numbers.

10. The numbers $a, b, 8,5,7$ have mean 6 and variance 2. Find the values of and b , if $a > b$. Hence find the semi interquartile range.

11. Given the information below showing the test marks.

Marks	10	15	20	22	23

Number of students	1	5	12	2	4

Calculate:

a) Mean

b) Variance

c) Mode

d) Semi interquartile range.

(ans:

12. The table below shows the test marks of a French paper marked out of 6.

Marks	1	2	3	4	5	6
Number of students	10	15	12	38	13	12

Calculate:

a) Mean

b) Variance

c) Standard deviation

d) Interquartile range

(Ans:

13. Given the table below

Height (m)	2	4	5	7	10	16
No. of trees	8	4	3	5	4	8

Calculate:

a) Mean

b) Variance

c) Mode

d) Semi-interquartile range

(Ans:

GROUPED DATA

14. Given the table showing the heights(cm) of 400 children in a certain school

Height (cm)	<100	<110	<120	<130	<140	150	<160	<170
Cumulative frequency	0	27	85	215	320	370	395	400

a) Calculate:

i. Mean
ii. Median
iii. Mode
iv. Variance
v. Interquartile range
vi. 4^{th} decile and 7^{th} decile
vii. 60% percentile
viii. Middle 70% percentile
ix. Middle 20% percentile
x. 10 to 90 percentile range

b) Draw a cumulative frequency curve and use it to estimate the median and semi interquartile range.

15. The masses measured to the nearest kilogram of 50 boys are noted as below

Mass (kg)	<59.5	<64.5	<69.5	<74.5	<79.5	<84.5	<89.5
Cumulative % of frequency	0	4	16	40	68	88	100

a) Draw a histogram and use it to estimate the mode

b) Represent the above information on an orgive and use it to estimate

i. Median
ii. Interquartile range
iii. Middle 60% percentile

c) Calculate :

i. Mean
ii. Median
iii. Mode
iv. Variance
v. Interquartile range
vi. 4^{th} decile and 7^{th} decile
vii. 60% percentile
viii. Middle 70% percentile
ix. Middle 20% percentile
x. 10 to 90-percentile range.

16. Given the information below showing the performance of students in a given test.

Marks(X)	Number of student.
20 – 29	5
30 – 38	7
39	6
40 – 44	4
45 – 60	8
61 – 80	2

a) Calculate the mean, median and mode.
b) Calculate the approximate 56% confidence range for the mean marks of all students.
c) Represent the above information on an ogive and use it to estimate the semi inter quartile range.

17. Given the information below showing the heights of different trees in a certain forest.

Height(X)	Number of trees.
- 25	10
- 38	8
- 58	12
- 60	10
- 63	6
- 83	15

a) Calculate the variance hence determine the standard deviation.

b) Estimate the 98% confident interval for the mean height of tree.

c) Represent the above information on a histogram and use it to estimate the mode.

18. Given the information below is showing the mass of different students in a certain class.

Mass(X)	Number of trees.
- 25	10
- 38	8
- 58	12
- 60	10
- 63	6
- 83	15
-95	0

(a) Calculate the inter quartile range.

(b) Estimate the 99% confident interval for the mean height of tree.

(c) Represent the above information on a histogram and use it to estimate the mode.

19. Given the information below showing the heights of different trees in a certain forest.

Height(X)	Number of trees.
20 -	10
25 -	22
40 -	38
50 -	50
60 -	66
75 -	80

(a) Calculate the mean and variance.

(b) Estimate the 95% confidence limits for the mean height of trees.

(c) Represent the above information on a histogram and use it to estimate the mode.

20. Given the information below.

Temperature	10 – 15	15 – 20	20 – 30	30 - 50	50 - 55	55 – 60
Frequency density	1	2.4	3.0	2.0	2.2	0.2

a) Calculate the mode and variance.

b) Estimate the 88% confidence limits for the mean temperature.

c) Represent the above information on the cumulative frequency curve hence use it to estimate the middle 60% percentile.

21. The table shows the marks obtained by 100 candidates of one school in the national examination.

Marks	20 -	40 -	45 -	55 -	60 -	70 -	75 -	80 - 90
Frequency	8	12	20	25	16	10	5	4

a) Calculate the mean and standard deviation.

b) Represent the above information on a histogram hence use it to estimate the mode.

c) Construct 88% confidence interval for the mean mark of all candidates in the whole country.

22. The weight of students of a certain class in a certain school are as indicated in the data below.

Weight	Number of students
50 – 53	3
54 – 57	8
58 – 61	12
62 – 65	18
66 – 69	11
70 – 73	5
74 – 77	2
78 – 81	1

(a) Estimate the mean and standard deviation of the students' weight

(b) Construct the cumulative frequency curve and estimate

(i) The median

(ii) The number of students who weigh between 58.9Kg and 66.7Kg.

23. Below are the marks of students of a certain school in the national mathematics contest.

Marks	Number of student.
10 – 15	2
15 – 25	8
25 – 30	17
30 – 35	26
35 – 45	24
45 – 50	16
50 – 60	6
60 – 80	1

(a) Draw a histogram for the above data

(b) Calculate the;

(i) Modal mark,

(ii) Median mark,

(iii) Number of students who passed, given that the pass mark was 39.

C) Draw a cumulative frequency curve and use it to estimate the;;

i) range of marks of the middle 80% of the students,

ii) number of students who got 37 and above ,

iii) passmark if 30% of the students failed,

iv) number of students who got between 22% and 54%

v) probability that a student has a mark betwwen 24% and 38%

d) Use this school to estimate the 95% confidence limits for the mean mark of all students in the country.

24. Given the information below.

Temperature	10 – 18	18 – 20	20 – 34	34 - 50	50 - 55	55 – 60
Frequency density	2.0	2.6	3.2	1.8	2.0	0.1

a) Calculate the mode and variance.

b) Estimate the 80% confidence limits for the mean temperature.

c) Represent the above information on the histogram hence estimate the mode.

25. The table below shows the amount of current (I) at which the electric fuses blow when it passed through them.

Current(I)	25- <28	<29	< 30	< 31	< 32	< 33	< 34	< 35	< 40

No. of fuses	6	12	27	30	18	14	9	4	5

a) Determine the;

   (i) Mean

   (ii) Median

   (iii) Standard deviation

   (iv) p_{85}

b) Draw a histogram for the data and use it to estimate the mode

c) Draw an orgive and use it to estimate the inter quartile range.

26. Given the table below showing the weights of different patients in a certain hospital in the given month.

Weight(W)	Cumulative frequency
$0 \leq w < 19$	30
$20 \leq w < 29$	46
$30 \leq w < 39$	70
$40 \leq w < 49$	102
$50 \leq w < 59$	130
$60 \leq w < 69$	142
$70 \leq w < 79$	150

a) Draw a histogram for the data and use it to estimate the mode

b) Draw an orgive and use it to estimate t

   i) The semi inter quartile range.

   ii) The number of patients who weigh between 24.5kg and 54.5k

27. Given the table below showing the marks obtained in an interview of a certain job

Weight(W)	Cumulative frequency
$0 \leq x < 15$	33
$16 \leq x < 20$	46

$21 \leq x < 39$	77
$40 \leq x < 49$	100
$50 \leq x < 59$	132
$60 \leq x < 69$	142
$70 \leq x < 89$	154

a) Draw a histogram for the data and use it to estimate the mode

b) Draw an orgive and use it to estimate t

i) The semi inter quartile range.

ii) The number of students who obtained between 18 kg and 80.5 kg.

CORRELATION AND SCATTER DIAGRAMS

28. The course work grades ranging from A to G and examination marks of 8 candidates are given below.

Course work	Examination marks
A	92
C	75
D	63
B	54
F	48
C	45
G	34
E	18

a) Calculate:

i) The spearman's rank correlation coefficient.

ii) Comment on your answer at 5% level of significance whether there correlation between the two ways of awarding.

29. The table below shows the original marks of six candidates in two examinations.

Candidate	A	B	C	D	E	F

English (x)	38	62	56	42	59	48
History (y)	64	85	84	84	64	69

a) Calculate:

i) The spearman's rank correlation coefficient and comment on your answer.

ii) Represent the above data on a scatter diagram. Draw a line of best fit and hence estimate x when y=90 and y when x=80

30. The price of Matooke is found to depend on the distance (d) the market is away from the nearest town. The table below gives the average price of Matooke for markets around Kampala city.

d(km)	40	8	17	20	24	30	10	28	16	28
P(sh.)	120	160	140	130	135	125	150	130	145	125

i) Plot this data on a scatter diagram.

ii) Draw a line of best fit on your diagram.

iii) Find the equation of your line in the form of $p = a + Bd$ where a and B are constants. Hence estimate the price of matooke when $d = s$.

31. In a certain commercial institution, a speed and error typing examination was administered to 12 randomly selected candidates A,B,C……L of the institution. The table below shows their speeds(y) in seconds and number of errors in the typed script (x)

	A	B	C	D	E	F	G	H	I	J	K	L
X	12	24	20	10	32	30	28	15	18	40	27	35
Y	130	136	124	120	153	160	155	142	145	172	140	157

i) Plot the data on a scatter diagram.

ii) Draw the line best fit on your diagram and comment on the likely association between speed and error made.

iii) Determine the equation of your line in the form $y = xk + b$ where k and b are constants.

iv) By giving Rank 1 to the fastest student and the student with the fewest errors, rank the above data and use it to calculate the correlation coefficient. Comment on your results.

32. The weighing scales from three different stalls W,X and Y in Owino market were used to weigh 10 bags of beans A,B,C………J and the results (in Kgs) were as given in the table below.

	A	B	C	D	E	F	G	H	I	J
Scale. W	65	68	70	63	64	62	73	75	72	78
Scale. X	63	68	68	60	65	60	72	70	70	66
Scale. Y	63	74	78	73	64	73	79	67	67	79

a) Determine the rank correlation, for the performance of scale

i) w and x

ii) x and y

b) Which of the three scales: W, X and Z were in good working conditions.

33. Given the information below showing the preferences of senior six boys and girls with respect to the meals they want to take during lunch

Girls	B	A	D	F	E	C	G
Boys	F	A	B	E	C	G	D

Calculate the spearman's rank correlation coefficient and comment on your answer.

34. Given the information below showing the preferences of secondary school boys and girls with respect to the meals, they want to take during the dinner.

Girls	G	A	D	B	G	E	G
Boys	D	G	B	F	C	A	E

Calculate the spearman's rank correlation coefficient and comment on your answer

35. Given the information below showing the preferences of secondary school boys and girls with respect to the meals, they want to take during the dinner.

Girls	G	B	D	A	G	E	G
Boys	D	G	B	F	C	A	E

Calculate the spearman's rank correlation coefficient and comment on your answer.

36. The table below the percentage of sand in soil (y) at different depth (x) in cm

Soil depth (x)	35	65	55	25	45	75	20	90	51	60
Percentage of sand (y)	86	70	84	92	79	68	96	58	86	77

a) (i) Calculate the rank correlation coefficient between the two variables

(ii) Comment on the significance at 5% level.

b) (i) Draw a scatter diagram for the data and comment on your result.

(ii) Draw the line of best fit hence estimate the;

- Percentage of sand in the soil at the depth of 30cm,
- Depth of soil with 54% sand.

37. Given the information below showing the preferences of secondary school boys and girls with respect to the examination body, they should do.

Girls	M	T	S	P	K	G	A
Boys	K	M	A	S	T	G	P

Calculate the spearman's rank correlation coefficient and comment on your answer.

38. Given the information below showing the preferences of secondary school boys and girls with respect to the examination body, they should do.

Girls	Q	T	V	P	Z	G	A
Boys	Z	V	A	Q	T	G	P

Calculate the spearman's rank correlation coefficient and comment on your answer.

Number index.

39. In April 2010, the price of a kilogram of sugar was shs. 4300. In April 2013, the price was shs. 5500. Taking 2010 as the base year, find the price relative and comment on your answer.
40. In August 2011, the price of a kilogram of salt was shs. 600. In April 2015, the price was shs. 500. Taking 2011 as the base year, find the price index and comment on your answer.
41. The table indicates the price of one house has changed over the year 2010, 2012and 2013

Year	2010	2012	2013
Price(euro)	70,100	100,000	68,000

Taking 2010 as the base year, calculate the simple price indices for 2008 and 2009.

42. The table indicates the price of one company has changed over the year 2007, 2011 and 2015

Year	2007	2011	2015
Price(shs.)	700,00	1,000,000	1,300,000

Taking 2010 as the base year, calculate the simple price indices for 2011 and 2015 an comment on your answers.

43. In 2005, the price index of a commodity using 2001 as the base year was 112. In 2011, the index using 2005 as the base year was 85. What would have been the index in 2011, using 2001 as the base year? (ans; 95.2)
44. In 2007, the price index of sugar using 2004 as the base year was 120. In 2012, the index using 2007 as the base year was 90. What would have been the index in 2012, using 2004, as the base year? (ans;)
45. In 2006, the price index of sugar using 2003 as the base year was 88. In 2010, the index using 2006 as the base year was 130. What would have been the index in 2010, using 2003, as the base year? (ans;)
46. The 2011 price index for a pair of shoes was 120 taking 2007 as the base year. Calculate the 2007 index taking 2011 as the base year.
47. The 2016 price index for a pair of shoes was 150 taking 2013 as the base year. Calculate the 2013 index taking 2016 as the base year.

48. The price relative of the commodity in 2001 using 2002 as the base year was 105. The price relative of the same commodity in 2002, using 2001 as the base year was 95. Given that the cost of the commodity in 2000 was 50,000, find its cost in 2002.

49. The below shows the price of items in Uganda shillings of some commodities as shown below;

Item	2008 price	2010 price
Rice(1 kg)	2500	3100
Sugar(1kg)	3000	3400
Eggs (1 dozen)	4000	4500
Groundnuts	2100	2600

Taking 2008 as the base year, Calculate;

a) The price index of each commodity,

b) Simple aggregate price index and comment on your answer.

50. The below shows the price of items in shillings of some commodities as shown below;

Item	2008 price	2010 price
Rice(1 kg)	2500	2000
Sugar(1kg)	3000	2950
Eggs (1 dozen)	4000	3600
Groundnuts	2100	1960

Taking 2008 as the base year, Calculate;

c) The price index of each commodity,

d) Simple aggregate price index and comment on your answer.

51. Given the table below showing the price relatives and their corresponding weights.

	Price relative	Weight
Food	118	40
Rent	102	8
Clothing	114	12

Fuel	120	10
Other	110	30

Calculate the weighted price index and comment on your answer.

52. Given the information below;

Item	2007price(euro)	2009(euro)	weight
Food	55	60	4
Housing	48	52	2
Transport	16	20	1

Calculate;

a) Price relative for each commodity.

b) Weighted price relatives (composite index)

c) Average weighted price index

53. The table below shows the expenditure of restaurant on four items and their simple index numbers for 2017

Item	Price 2015	Price 2017	Index numbers	Weight
Milk(per litre)	1000	A	130	0.5
Eggs (per tray)	B	9000	125	1
Sugar (per kg)	3000	3800	C	2
Blue band	7000	D	130	1

(a) Determine the values of **a, b, c** and **d**.

(b) Calculate the weighted aggregate price index for 2017. Comment on your result.

(c) In 2017, the restaurant spent shs. 43,000 on buying these items, how much money the restaurant could have spent in 2015.

54. The average cost of living (in '000s of shillings per month) in 1998 was:

Item	Cost ('000s Ug Sh.)
Water	10.0
Fuel	12.0
Clothing	43.0
Transport	55.0
School fees	300.0
Medical	30.0
Food	100.0
Miscellaneous	50.0
Saving	100.0

a) Taking water as the base item, calculate the cost of living index of other items in 1998

b) If the family income increased by shillings 100000, while the ratio of their budget remained the same in 1999, construct a table to show the monthly cost of living for 1999

55. The table below shows retail prices of clothing and foot wear (in '000s Ug Sh) for may 1996, june 1997 and price relatives for july 1998

Item	May-96	Jun-97	Price relatives for july 1998(may 1996 as base)
Men's clothing	70.0	70.0	1.2
Women's clothing	60.0	80.0	1.5

Children's clothing	30.0	130.0	1.5
Foot wear	50.0	35.0	0.8
School fees	300.0	60.0	0.8

i) Using children's clothing as the base, find the indices of other clothing and foot wear for May 1996

ii) Using June 1997 as the base year, calculate the retail prices for 199

56. Given the table below showing price of different items as shown below.

Item	Price in 2010	Price in 2016	Amount consumed
Sugar (1 kg)	\$2	\$3	4
Salt(1kg)	Ug. Sh. 400	Ug. Sh. 600	1
Maize flour(1 kg)	\$0.5	\$ 1	7

Calculate the weighted aggregate price index

57. Given the table below showing price of different items as shown below.

Item	Price in 2010	Price in 2016	Amount consumed
Rice (1 kg)	\$3	\$6	4
Salt(1kg)	Ug. Sh. 500	Ug. Sh. 700	2
Maize flour(1 kg)	\$0.5	\$ 2	2

Calculate the weighted aggregate price index.

BEGINNING PROBABILITIES AND PROBABILITY THEORY.

1. Events A and B are such that $P(A) = 0.35, P(B) = 0.7$ and $P(AnB') = 0.15$. find the:

   i) $P(^{A}/_{B})$

   ii) $P(A/B')$

   iii) $P(A'/B')$

   (Answer: i) $\frac{2}{7}$ ii) $\mathbf{0.5}$ iii) $\mathbf{0.5}$)

2. In a certain school, the probability that a boy chosen randomly belongs to the football team is 0.6 and that f chess is 0.5. The probability that he belongs to at least one of the teams is 0.9. Find the probability that he belongs to either football team or the chess team but not both. ($\mathbf{Answer. 0.7}$)

3. a) Given that A and B are independent events and that A' and B' are respectively their complements.

   i) Prove that A' and B' are also independent.

   ii) If P(A)=0.5 and P(B)=0.4, find the $P(A'nB')$

   c) Bag A contains 4 red and 6 green sweets. Bag B contains 3 red and 7 green sweets. Bag A is twice as likely to be picked as bag B. A bag is randomly selected and from it, a sweet is picked randomly and put into the other bag. A sweet is then picked from the latter bag. Find the probability the sweet picked from the bag is red.

   ($\mathbf{Answer.\ aii.\ 0.3,\ b).\ \frac{37}{110}}$)

4. A and B are two independent events with A twice as likely to occur as B. if P(A)=$\frac{1}{2}$, find the:

   i) P (A or B but not both)

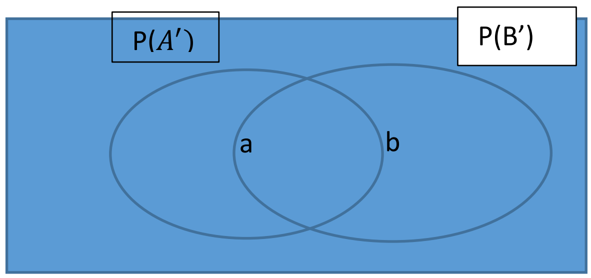
   ii) P $(^{A}/_{B'})$

   ($\mathbf{Answer.\ i.\ 0.5,\ ii)\ 0.5}$)

5. One bag of chocolates contains contains five hard centered and three soft centered chocolates. Another bag of chocolates contains eight hard centered and seven soft centered. A chocolate is chosen at random from either of the bags. Find the probability that a soft centered chocolate came from the first bag.

$\left(\textbf{Answer.} \frac{3}{16} \right)$

6. A' and B' are intersecting sets as shown in the venn diagram below



Given that A' and B' are complements of A and B respectively and that P(A)=0.8, P(B)=0.6 and P(either A' or B' but or both)=0.45. find the values of **a** and **b**.

(Answer: $a = 0.475$,

7. A man travelling from Masaka to Kampala by private car goes through three police check points. The probability that he is delayed by checking point A is 0.3, for checking point B and C is about 0.5 and 0.7 respectively. Find the probability that;

   i) He is not delayed at all.

   ii) He is delayed at only one checking point.

(Answer: i) 0.105, ii) 0.250)

8. Events A and B are such that $P(A/B) = \frac{1}{3}, P(B/A') = \frac{5}{8}$, $p(B) = 0.1$ and $P(A' \cap B') = \frac{3}{20}$. Find ;

   i) $P(A \cap B')$

   ii) $P(A'/B')$

(Answer: i) $\frac{19}{40}$, ii) $\frac{1}{6}$)

9. A box contains 6 black, 5 red and 4 green balls. Three balls are picked at random one at a time without replacement. Find the probability that;

i) All the balls picked are of the same color.

ii) The third ball picked is the second black ball to be picked.

$$\text{(Answer: i)} \frac{34}{455}, \text{ii)} \frac{18}{91} \text{)}$$

10. If events X and Y are independent.

a) Show that even X' and Y are also independent.

b) Find $P(X\cap Y')$ given that $P(X') = \frac{3}{4}a$ and $P(Y)=\frac{2}{5}$.

(Answer: b) $\frac{3}{20}$)

11. Dan's probabilities of passing Biology, Chemistry and Mathematics are 0.6, 0.75 and 0.8 respectively.

a) Find the probability that he will pass at least two subjects.

b) If it is known that he passed at least two subjects, what is the probability that he failed chemistry?

(Answer: a) 0.45 b) 0.2)

12. Events A and B are such that $P(A)=\frac{4}{7}$, $P(A \cap B') = \frac{1}{3}$ and $P(A/B) = \frac{5}{14}$. Find ;

i) $P(A \cap B)$

ii) $P(B)$

iii) $P(A \cup B)$

iv) $P(A' \cup B')$

v) $P(A' \cap B')$

(Answer: i) $\frac{5}{21}$, ii) $\frac{2}{3}$, iii 1, iv) $\frac{16}{21}$, v) 0)

13. Given that $p(A/B) = 0.4$, $p(A/B') = 0.5$ and $P(A) = 0.3$. Find $P(A)$ and $P(A \cup B)$.

(**Answer**: $\mathbf{p(B) = 0.5\ p(\ A\ U\ B) = 0.6}$)

14. Given that $p(^{A}/_{B}) = 0.5, p\left(^{A}/_{B'}\right) = 0.35$ and P (A) = 0.38. Find P(A) and P(A U B).

    (**Answer**: $\mathbf{p(B) = 0.2,\ \ p(\ A\ U\ B) = 0.48}$)

15. Given that $p(^{B}/_{A}) = 0.4, p\left(^{B}/_{A'}\right) = 0.55$ and P (B) = 0.45. Find P(A) and P(A U B).

    (**Answer**: $\mathbf{p(B) = \frac{2}{3}\ p(\ A\ U\ B) = 0.85}$)

16. Given that $p(^{B}/_{A}) = 0.6, p\left(^{B}/_{A'}\right) = 0.5$ and P (B) = 0.55. Find P(A) and P(A U B).

    (**Answer**: $\mathbf{p(B) = 0.5\ \ p(\ A\ U\ B) = 0.75}$)

17. A and B are two independent events. If $P(A) = {}^{1}/_{2}\ and\ P(B) = {}^{1}/_{4}$
    Find
    (i)$P(\ \mathrm{Au}B^{1})$
    (ii)P $\left(^{AnB}/_{A}\right)$

18. In a car assembly plant, machines A,B and C produce 37%, 42% and 21% respectively of the total production. If 0.6% of the production form A is defective and that from B and C are 0.4% and 1.2%, find the probability that a car selected at random from the plant;
    i) Is defective
    ii) Came from C given that it is defective.

    (**Answer**: $\mathbf{i)\ 0.00642,\ \ ii)\ \frac{42}{107}}$)

19. Two events A and B are independent such that P(A)=0.75P(B), P(A ∪ B) $= \frac{7}{10}$, calculate the;
    i) P(A)
    ii) P(AnB)
    iii) $P(^{B}/_{A})$

    (**Answer**: $\mathbf{i)\ 0.3845\ ,\ \ ii)\ 0.1971, iii)\ 0.5126}$)

20. A and B are independent events with $P(A) = \frac{3}{8}$ and $P(A' \cup B) = \frac{3}{4}$. Find;

i) $P(B)$

ii) $P(A \cup B)$

$$Answer\ i)\ \frac{1}{3} \quad ii)\ \frac{7}{12}$$

21. A box P contains 1 red, 3 green and 1 blue beads. A box Q contains 2 red, 1 green, and 2 blue bead. A balance die is thrown and if the throw shows a six, box P is chosen otherwise box Q is chisen. A bead is drawn at random from the chosen box. Given that a green bead is drawn, find the probability that it came from box P?

$Answer\ \frac{3}{8}$

22. A and B are mutually exclusive. If $P(A \cup B) = \frac{3}{4}$ and $P\left({A'}/{B'}\right) = \frac{2}{3}$; find;

i) $P(A)$

ii) $P(B)$

$$Answer\ i)\ \frac{5}{8}\ \ ii)\ \frac{5}{8}$$

23. A bag contains 5 black marbles and 3 white marbles. A second bag contains 3 black marbles and 5 white marbles. A marble is drawn at random from the first bag and placed in the second bag. A marble is then drawn at random from the second bag and placed in the first.Find the probability that each bag now contains;

i) 4 black and 4 white marbles

ii) Exactly the same number of each colour as it did initially.

$$Answer\ i)\ \frac{25}{72}\ ii)\ \frac{19}{36}.$$

24. Bag A contains 6 black marbles and 7 white marbles. A second bag B contains 4 black marbles and 8 white marbles. A marble was drawn at random from bag A and placed in $\boldsymbol{B}$. A marble was then drawn at random from B into A.

a) Find the probability that A contains exactly the same number of marbles as it had initially.

b) Find the probability of picking a black marble now from A

c) Find the probability of picking a white marble now from A

$Answer\ i)\qquad ii)$

25. A bag contains 4 white balls and 1 black ball. A second bag contains 1 white ball and 4 black balls. A ball is drawn at random from the first bag and put into the second bag, then a ball is taken from the second bag and put into the first bag.
    a) Find the probability that the first bag is containing the same number as it had before.
    b) Find the probability that a white ball will be picked when a ball is selected from the first bag.

    $Answer\!:\ a)\qquad b)\ 0.7$

26. A bag contains 5 red balls and 4black ball. A second bag contains 4 red ball and 7 black balls. A ball is drawn at random from the first bag and put into the second bag, then from a ball from ball is taken from the second bag and put into the first bag.
    c) Find the probability that the first bag is containing the same number of ball as it had before.
    d) Find the probability that a red ball will be picked when a ball is selected from the first bag.

    $Answer\!:\ a)\qquad b)$

27. A box A contains 3 red balls and 4 black balls while box B contains 3 red and 2 black balls. One box is selected at random and from it, one ball is selected at random.
    a) Find the probability that the ball is red,
    b) The probability that the ball came from box A, given that it is red,
    c) The probability that the ball came from box $\boldsymbol{B}$, given that it is red,

    $Answer\!:\ a)\qquad b)$

28. A box A contains 6 red balls and 5 black balls while box B contains 4 red and 3 black balls. One box was selected at random and from it, two balls were selected at random.
    i) Find the probability that the balls are of the same color,
    ii) The probability that the ball came from box A, given that second ball is red,

iii) The first ball is red given that the second is black,

iv) The first is black given that the second is also black.

$Answer$: a) b)

29. A box A contains 5 red balls and 5 green balls while box B contains 4 red and 3 green balls. One box was selected at random and from it, two balls were selected at random. Given that the probability of selecting box A is twice that of selecting of B.
    a) Find the probability that the balls are of different color,
    b) The probability that the ball came from box B, given that second ball is red,
    c) The first ball is red given that the second is green,
    d) The first is green given that the second is also green.

$Answer$: a) b)

30. A box A contains 7 red balls and 5 green balls while box B contains 4 red and 4 green balls. One box was selected at random and from it, two balls were selected at random. Given that the probability of selecting box A is thrice that of selecting of B.

i) Find the probability that the balls are of different color,

ii) The probability that the ball came from box B, given that second ball is green,

iii) The first ball is red given that the second is green,

iv) The first is green given that the second is also red.

$Answer$: a) b)

31. A box P contains 7 red balls and 5 green balls while box Q contains 4 red and 4 green balls. One box was selected at random and from it, two balls were selected at random. Given that the probability of selecting box Q is thrice that of selecting of P.
    a) Find the probability that the balls are of same color,
    b) The probability that the ball came from box P, given that second ball is green,
    c) The first ball is green given that the second is red,

d) The first is red given that the second is also red.

$Answer$: $a)$ $b)$

32. Given that $P(A) = 0.4, P(B) = 0.3, P(C) = 0.5, P(A \cap C) = 0.25$ and $P(A \cap B \cap C) = 0.003$. if A and B as well as B and C are mutually exclusive, find the probability of obtaining;

    i) A or B or C,
    ii) A or B,
    iii) B or C.

    $Answer$: $a)$ $b)$

33. Given that $P(A) = 0.2, P(B) = 0.4, P(C) = 0.6, P(A \cap C) = 0.45$ and $P(A \cap B \cap C) = 0.1$. if A and B are independent while B and C are mutually exclusive, find the probability of obtaining;

    i) A or B or C,
    ii) A or B,
    iii) B or C.

    $Answer$: $a)$ $b)$

34. Given that $P(A) = 0.12, P(B) = 0.45, P(C) = 0.55, P(A \cap C) = 0.5$ and $P(A \cap B \cap C) = 0.1$. if A and B are independent while B and C are mutually exclusive, find the probability of obtaining;

    a) A or B or C,
    b) A or B,
    c) B or C.

    $Answer$: $a)$ $b)$

35. A bag contains 5 green, 10 blue and 6 red marbles. Three marbles were picked at random without replacement. Find the probability that;

i) The first marble is blue and the third marble is green.

ii) The first marble is green and the third marble is green.

iii) The first marble is blue and the third marble is red.

iv) The first marble is red and the third marble is green.

$Answer$: a) b)

36. A bag contains 20 black, 30 blue and 50 red marbles. Three marbles were picked at random without replacement. Find the probability that;

a) The first marble is blue and the third marble is black.

b) The first marble is red and the third marble is black.

c) The first marble is blue and the third marble is red.

d) The first marble is red and the third marble is black.

$Answer$: a) b)

37. A bag contains 7 green, 15 blue and 6 red marbles. Three marbles were picked at random without replacement. Find the probability that;

i) The first marble is blue and the third marble is green.

ii) The first marble is green and the third marble is green.

iii) The first marble is blue and the third marble is red.

iv) The first marble is red and the third marble is green.

$Answer$: a) b)

38. A bag contains 4 black marbles and 10 white marbles. A second bag contains 5 black marbles and 4 white marbles. A marble is drawn at random from the first bag and placed in the second bag. Given that, the probability of choosing the first bag is thrice that of the second bag.

a) Find the probability that the first bag contains the same number of marbles as it had initially.

b) Find the probability of picking a white marble now.

$Answer$ i) ii)

39. Events A and B are such that $\mathrm{P}(^{\mathrm{A}}/_{\mathrm{B}}) = 0.4, \mathrm{P}(\mathrm{A}) = 0.2$ and $\mathrm{P}(\mathrm{B}) = 0.25$. find;

i) $P(A \cap B)$

ii) $P(A \cup B)$

iii) $P(A'/B)$

$Answer\ i)\ 0.1\ ii)\ 0.350\ iii)\ 0.60$

40. Events X and Y are independent such that P(X)=0.75 and $P(X \cup Y)' = \frac{1}{6}$. Calculate;

i) $P(Y)$

ii) P(X or Y but not both)

$Answer\ i)\ \frac{1}{3}\ ii)\ \frac{7}{12}$

41. Given that S and T are mutually exclusive events with P(S) $=\frac{2}{3}$ and P (T) =0.2.

find;

i) $P(S \cap T')$

ii) $P(S' \cup T')$

$$Answer\ i)\ \frac{2}{3}\ ii)\ 1$$

42. a) Prove that for any two events A and B; P (A/B) +P (A'/B) =1.

c) Find $P(A \cap B)$, given that $P(A) = \frac{9}{35}, P(B) = \frac{1}{15}$ and $P(A \cup B) = \frac{4}{15}$

$$Answer\ \frac{2}{35}$$

43. Two events A and B are such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. find:

i) P (A'/B')

ii) $P(A \cup B)$

$$Answer\ i)\ \frac{7}{12}, ii)\ \frac{17}{24}$$

44. Events A and B are such that P(A)=0.7, P(B)=0.2 an $P(A/B) = 0.1$, find the probability that;

a) Both of the events will occur

b) Exactly one of the events occur.

$$Answer\ a)\ \ 0.02, b)\ 0.86$$

45. The probability that it will be foggy on the November morning is $\frac{1}{3}$. The probability that Mr. John will be late for work if it is not foggy is $\frac{1}{8}$ otherwise it is $\frac{4}{5}$.

a) Find the probability that he is late.

b) If on a particular November morning Mr. Jones is late, find the probability that it is foggy.

$Answer\ a)\ \frac{7}{20}\ b)\ \frac{16}{21}$

46. Three boys; Paul, Robert and Maurice aim at a target. The probability of them hitting the target are 0.8, 0.7 and 0.6 respectively. Find the probability that;

i) All the three hit the target,

ii) Exactly two of them hit the target.

$$Answer;\ \ i)\ 0.336\ ii)\ 0.452$$

47. Events A and B are such that $\mathrm{P(A)} = \frac{4}{7}$, $\mathrm{P(AnB')} = \frac{1}{3}$ and $\mathrm{P}\left({}^{\mathrm{A}}/_{\mathrm{B}}\right) = \frac{4}{5}$. Find;

a) P(B)

b) P(A’nB’)

$$Answer;\ \ a)\ \frac{25}{84}\ b)\ \frac{31}{84}$$

48. Events A,B and C are such that $\mathrm{P(A) = x, P(B) = y}$ and $\mathrm{P(C) = x + y}$. If $\mathrm{P(A \cup B) = 0.6}$ and $\mathrm{P}\left({}^{\mathrm{A}}/_{\mathrm{B}}\right) = 0.2$.

i) Show that $5\mathrm{x} + 4\mathrm{y} = 3$

ii) Given B and C are mutually exclusive and that $\mathrm{P(B \cup C) = 0.9}$, determine the values of x and y

$$\boldsymbol{Answer\ x = 0.4\ and\ y = 0.25}$$

49. According to the firm’s internal survey, of those Employees living more than 2 miles from work, 90% travel to work by car, of the remaining employees, only 50% travel to work by car. It is known that 75% of the employees live more than 2 miles from work.

Determine;

i) The overall proportion of employees who travel to work by car.

ii) The probability that an employee who travels to work by car lives more than 2 miles from work.

iii) The probability that an employee who lives more than 2 miles travels to work by car.

Answer $\mathbf{i)\ 0.8\ ii)\ 0.9\ iii)\ 0.8438}$

50. If $P(A/B) = \frac{2}{5}, P(B) = \frac{1}{4}, P(A) = \frac{1}{3}$,find;

i) P(A/B’)

ii) $P(A' \cup B')$

Answer $i)\ \frac{14}{45}\ , ii)\ \frac{9}{10}$

51. A company has four production lines A, B, C and D producing large numbers of a certain item. Of the total daily production, 40% are produced by A, 20% by B, 15% by C and the rest by D.it is known that 2% of the items produced by A are defective. The corresponding proportions for B,C and D are 3%,4% and 1% respectively. One item is chosen from the day’s total output.

i) Find the probability that it is not defective.

ii) Given that it is defective, find the probability that it was produced by A.

Answer $\mathbf{i)\ 0.9775\ ii)\ \frac{16}{45}}$

52. Given that X and Y are independent events and that X’ and Y’ are respectively there complements,

i) Prove that X’ and Y’ are also independent.

ii) If P(X)=0.5 and P(Y)=0.4, find P(X’nY)

Answer $\mathbf{ii)\ 0.2}$

53. The staff employed by a certain school is classified as Academic, Administrative or Support. The table below shows the numbers employed in these categories and there sex

	Male	Female
Academic	70	10
Administrative	8	2

Support	29	6

If a member of staff is selected randomly, find the probability that;

i) She is female

ii) The staff member is either female or Academic but not both

iii) The administrative staff member selected is male.

54. (a) Given events A and B are independent such that 3P (A∪ B)=5P (B)=4P (A). find

i. P(A)

ii. $P(A\cup B')$

(b) Given events A and B are such that $P(A\cup B) = 0.8, P({}^{A}/_{B}) = 0.3$ and $P(A' \cap B) = 0.4$. find;

i. $P(A\cap B)$

ii. P(B)

iii. $P(A^1)$

iv. $P({}^{A}/_{B'})$

$$Answer\ ai)\ \frac{7}{12}\ \ aii)\ \frac{7}{9}$$

55. Jane, Joan and James are playing a game in turns with Joan first followed by Jane and James respectively. The probabilities of Jane Joan and James winning are 0.1, 0.3 and 0.15 respectively. Find the probability of that

i. Jane wins on the second chance.

ii. James wins.

56. John is out of school and his probability of coming back is 0.4. The probability that is back and his team win the game is 0.82 otherwise its 0.35. Find the probability that they lose, given that he is a round.

57. A, B and C are three events completing all together with their respective probabilities of winning as $0.4, 0.6\ and\ 0.7$. Find the probability that;

a) All of them win,

b) Only one wins,

c) Only two win,

d) None of them wins.

58. P, Q and C are three events completing all together with their respective probabilities of winning as $0.1, 0.35$ *and* 0.45. Find the probability that;
    i) All of them win,
    ii) Only one wins,
    iii) Only two win,
    iv) None of them wins.
59. A, B and C are three events completing all together with their respective probabilities of winning as $\frac{1}{2}$, $\frac{2}{3}$ *and* $\frac{2}{5}$. Find the probability that;
    a) All of them win,
    b) Only one wins,
    c) Only two win,
    d) None of them wins.
60. A, B, C and D are four events completing all together with their respective probabilities of winning as $\frac{1}{2}$, $\frac{2}{3}, \frac{1}{6}$ *and* $\frac{2}{5}$. Find the probability that;
    i) All of them win,
    ii) Only one wins,
    iii) Only two win,
    iv) Only three win,
    v) None of them wins
61. A, B and C are three events completing all together with their probabilities of not winning are $0.8, 0.88$ *and* 0.5 respectively. Find the probability that;
    a) All of them win,
    b) Only one wins,
    c) Only two win,
    d) None of them wins.

62. A, B and C are three events completing in turns with their respective probabilities of winning as $0.4, 0.6$ *and* 0.7. Given that A starts first followed by B and finally C, find the probability that;

a) B wins in the first trial,

b) C wins on the second trial,

c) A wins on the third trial,

d) A wins,

e) B wins.

63. A, B and C are three events completing in turns with their respective probabilities of winning as $0.12, 0.66$ *and* 0.78. Given that C starts first followed by A and finally B, find the probability that;

i) C wins on the second trial,

ii) A wins on the four trial,

iii) B wins.

64. A, B, C and D are three events completing in turns with their respective probabilities of winning as $0.12, 0.66 \ \ 0.3$ *and* 0.78. Given that D starts first followed by A , then C and finally B, find the probability that;

a) C wins on the second trial,

b) A wins,

c) B wins.

65. A, B and C are three events completing in turns with their respective probabilities of not winning are $\frac{1}{4}, \frac{2}{5}$, *and* $\frac{1}{3}$. Given that B starts first followed by A and finally C, find the probability that;

iv) C wins on the second trial,

v) A wins on the four trial,

vi) B wins.

66. Mr Jacob is the senior inspector in an electronics-manufacturing firm. One of his roles is to inspect in coming lots of memory sticks produced by two terminals daily and determine their effectiveness. In a tray containing 12 sticks, four were found to be defective. He randomly selects three sticks for inspection. Form a probability distribution showing the number of defective sticks. Hence, find;

i. Probability that at most two of the three is non-defective

ii. The expected number of non-defective sticks

(b) A petrol station served three times as many men as women on a certain day. Two types of fuel are available, leaded (L) and unleaded (U). Customers pay by cheque or by cash. 40% of the men and 80% of the women buy the leaded type. Of the men buying L 60% pay by cheque and of the men buying U, 70% pay by cheque. Of the women buying L, half pay by cheque while those buying U, 30% pay by cheque. Find the probability that a customer;

i. Buys leaded fuel.
ii. Pays by cheque
iii. Who pays by cheque for L is a man.

67. The table below shows the number of apples in boxes A, B and C

Apples	A	B	C
Green	5	7	4
Blue	6	5	4

A box is randomly selected and two apples are randomly selected from it without replacement. Box A is thrice as likely to be picked as B while A and C have the same chance of being selected.

(a) Determine the probability that;

(i) All apples are of different colors,

(ii) From box A given that they are of the same color.

(b) If X is the number of green apples taken, draw a probability distribution table for X. hence calculate the mean and standard deviation of X.

$Answers\ ai)\ 0.556\ \ aii)\ 0.429\ b)\ \ 1.021, 0.666$

68. The table below shows the number of apples in boxes A, B and C

Apples	M	N	F
Red	3	6	5
Blue	6	5	4

A box is randomly selected and two apples are randomly selected from it without replacement. Box M is twice as likely to be picked as N while M and F have the same chance of being selected.

a) Determine the probability that;
   (i) All apples are of different colors.
   (ii) From box N given that they are of the same color
b) If X is the number of blue apples taken, draw a probability distribution table for X. hence calculate;
   i) The expected number of blue apples (X),
   ii) The most likely number of blue apples,
   iii) The variance of blue apples,
   iv) If the cost of each apple is $y = 100X + 200$, find var(y).

$$Answer\ ai)\ \frac{263}{495}\quad aii)\ \frac{45}{232}\quad bi)\ \frac{106}{99}\quad bii)\ 1\quad biii)\ 0.464\quad biv)\ \ 4640.$$

69. The discrete random variable X has p.d.f $P(X = x)$ for $x = 1,2,3$.

X	1	2	3
P(X=x)	0.2	0.3	0.5

Find:

a) $E(X)$

b) $E(X^2)$

c) $Var(X)$

d) $Var(\ 4x + 5)$

$$(\ Answer;\ \ a)\ \ 2.3,\qquad b)\ 5.9,\ \ c)\ 0.61,\qquad d)\ \ 9.76)$$

70. The discrete random variable X has the probability distribution specified in the following table.

X	-1	0	1	2
P(X=x)	0.25	0.10	0.45	0.20

a) Find $P(-1 \leq X < 1)$
b) Find $E(2X + 3)$

c) Sketchf(x) and F(x).

d) Find F(x) hence use it to find;

i) $P(0 \le X \le 1)$

ii) $p(X < 1)$

iii) $p(X > 1)$

$(\,Answer;\ a)\ 0.35, \quad b)\ 4.2, \ di)\ 0.55, \quad dii)\ 0.35 \quad diii)\ 1.0 \ )$

71. The random variable X has p.d.f $P(X = x)$ as shown in the table

X	-2	-1	0	1	C
P(X=x)	0.1	0.1	0.3	0.4	0.1

Find the value of c(a) if $E(X) = 0.3$, (b) if $E(X^2) = 1.8$

$(\,Answer;\ a)\ 3, \quad b)\ 3\ )$

72. Given $p(X \le x) = \beta(x+1)$, for $x = 0, 1, 2, 3, 4$. Find:

a) The value of β

b) $Var(2x - 90)$.

$(\,Answer;\ a)\ \frac{1}{5}, \quad b)\ \ 8\ )$

73. The discrete random variable X has a probability function given by

$$P(x) = \begin{cases} \left(\frac{1}{2}\right)^x & x = 1,2,3,4,5 \\ c & x = 6 \\ 0 & \text{otherwise} \end{cases}$$

Where c is a constant.

a) Determine the value of c and hence the mode and mean of X

b) Median

c) Sketch of f(x) and F(x).

$(\,Answer;\ a)\ \frac{1}{32}, 1, \frac{63}{32} \ \ b)\ 1\ )$

74. The discrete random variable X has a p.d.f

$$P(X = x) = k|x|$$

Where x takes the values -3,-2,-1, 0,1,2,3.

Find:

a) The value of the constant k

b) E(X)

c) Var(x) and the standard deviation of X

d) $p(|x| \leq 1)$

e) $p(|x| \geq 1)$

(Answer: a) $\frac{1}{12}$ b) 0 c) 6, $\sqrt{6}$ d) $\frac{1}{6}$, e) $\frac{5}{6}$)

75. The discrete random variable X has a distribution function F(x) where

$F(x) = 1-(1-\frac{1}{4}x)^x$ for x=1, 2, 3, 4.

a) Show that $F(3) = \frac{63}{64}$ and $F(2) = \frac{3}{4}$.

b) Obtain the probability distribution of X

c) Find E(X) and Var(X)

d) Find $P(X > E(X))$

(Answer: c) $2\frac{1}{64}$, 0.547 d) $\frac{1}{4}$)

76. A discrete random variable X has a probability mass function

$$f(x) = \begin{cases} Kx, & x = 1, 2, 3, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

Where K is a constant

a) Show that $K = \frac{2}{n(n+1)}$,

b) Find in terms of n, the mean of X

c) Find the variance of X if n is 5

$$(\textit{Answer};\ b)\ \frac{(2n+1)}{3}, c)\ \frac{14}{9})$$

77. A discrete random variable X has a probability mass function

$$f(x) = \begin{cases} \beta x, & x = 1, 2, 3, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

Where K is a constant and mean is 3

a) Find the values of β and n

b) Find the variance of X.

$$\left(\textbf{\textit{Answer}};\ \textbf{\textit{a}})\ \boldsymbol{n = 4},\ \boldsymbol{\beta = \frac{1}{10}},\ \textbf{\textit{b}})\ \mathbf{1}\ \right)$$

78. A discrete random variable X has a probability mass function

$$f(x) = \begin{cases} \alpha x, & x = 1, 2, 3, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

Where K is a constant and mean is 7

a) Find the values of α and n

b) Find the variance of X.

$$\left(\textbf{\textit{Answer}};\ \textbf{\textit{a}})\ \boldsymbol{n = 10}, \quad \alpha = \frac{\mathbf{1}}{\mathbf{55}},\right.$$

79. A discrete random variable X has a probability mass function

$$f(x) = \begin{cases} hx, & x = 1, 2, 3, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

Where K is a constant and mean is 9

i) ind the values of h and n

ii) Find the variance of X.

$$\left(\textbf{\textit{Answer}};\ \textbf{\textit{i}})\ \boldsymbol{n = 13},\ \boldsymbol{\beta = \frac{1}{91}},\ \textbf{\textit{ii}})\ \ \right)$$

80. For a discrete random variable X has the cumulative distribution function is given by $F(X) = \frac{x^2}{9}$ For $x = 1,2,3$

a) Find $F(2)$

b) Find $P(X = 2)$

c) Write down the probability distribution of X

d) Find $E(2X - 3)$

(Answers: a) $\frac{4}{9}$ **b)** $\frac{1}{3}$ **c)** $\mathbf{p(X = x) = \frac{2x-1}{9}, x = 1, 2, 3}$ **d)** $\frac{17}{9}$ **)**

81. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} \dfrac{kx}{(x^2+1)}, & x = 2,3 \\ \dfrac{2kx}{(x^2-1)}, & x = 4,5 \\ 0, & \text{otherwise} \end{cases}$$

a) Show that the value of k is $\frac{20}{33}$

b) Find the probability that X is less than 3 or greater than 4

c) Find $F(3.2)$

d) Find i) $E(X)$, ii) $Var(X)$

$(\boldsymbol{Answer};\ \boldsymbol{b})\ \frac{49}{99},\ \boldsymbol{c})\ \frac{14}{33}\ \ \boldsymbol{di})\ \mathbf{3}\ \frac{58}{99}, \boldsymbol{ii})\ \mathbf{1.23}\ )$

82. X is a discrete random variable such that. $P(X \le x) = k(X + 1)$. for $x = 1, 2, 3, 4, 5, 6$.

Find.

i. The pdf of x

ii. Variance of x

iii. Sketchf(x)andF(x).

iv. If $y = (4x - 3)$, find variance of y.

$(\boldsymbol{Answer};\ \boldsymbol{ii})\ \frac{63}{16},\ \boldsymbol{iv})\ \mathbf{63}\ )$

83. X is a discrete random variable such that. $P(X \leq x) = \mu(x^2)$. for $x = 1, 2, 3$.

a) Find the value of μ

b) Find the variance of X

(***Answer***; ***a***) $\mu = \frac{1}{10}$, ***b***) $\frac{17}{36}$)

84. A biased tetrahedron has faces numbered 1, 2, 3 and 4. If the die is tossed, the probability of the face is inversely proportional to the number on the face. If x is the random variable the number on the face the die lands on, determine:

i. Probability distribution for x
ii. Var (x)
iii. $P(|x - 1| \leq 1)$

(***Answer***; ***i***) $p(X = x) = \frac{12}{25x}$, $x = 1, 2, 3, 4.$, ***ii***) **1.1136** ***iii***) $\frac{18}{25}$)

85. Anne plays a game in which a fair six-sided die is thrown once. If the score is 1,2 or 3, Anne loses €10. If the score is 4 or 5, Anne wins €x. If the score is 6, Anne wins €2x.

a) Show that the expectation of Anne's profit is $\epsilon(\frac{2}{3}x - 5)$ in a single game.

b) Calculate the value of x for which, on average, Anne's profit is zero.

c) Given that $x = 12$, calculate the variance of Anne's profit in a single game.

(***Answer***; ***b***) **7.5**, ***c***) **185**)

86. A biased die has faces numbered 1, 2, 3 and 6. If this die is tossed, the probability of the face is directly proportional to the square of the number on the face. If x is the random variable the number on the face the die lands on, determine;

i. The probability distribution for x .
ii. Var(x)
iii. $P(|x - 3| \leq 1)$

(***Answer***; ***ii***) $\frac{20}{9}$, ***c***) $\frac{3}{7}$)

87. A discrete random variable X can take only the values 0, 1,2 or 3, and its probability distribution is given by $\mathrm{P(X = 0) = k, P(X = 1) = 3k, P(X = 2) = 4k, P(K = 3) = 5k}$, where k is a constant. Find

a) The value of k

b) The mean and variance of X

$$(\boldsymbol{Answer};\ \boldsymbol{a})\ \frac{\mathbf{1}}{\mathbf{13}},\qquad \boldsymbol{b})\ \mathbf{2}, \frac{\mathbf{12}}{\mathbf{13}}\ )$$

88. The random variable X is $B(\ 6, 0.42\ )$.find

a) $P(X = 6)$

b) $P(X = 4)$

c) $P(X \leq 2)$

$(\boldsymbol{Answer};\ \boldsymbol{a})\ \mathbf{0.00549}\ \boldsymbol{b})\ \mathbf{0.1570}\ ,\ \boldsymbol{c})\ \mathbf{0.503}\ )$

89. An unbiased die is thrown seven times. Find the probability of throwing at least 5 sixes

$(\boldsymbol{Answer};\ \mathbf{0.00200})$

90. A fair coin is tossed six times. Find the probability of throwing at least four heads.

$(\boldsymbol{Answer};\ \mathbf{0.3440})$

91. Assuming that a couple are equal likely to produce a boy or a girl, find the probability that in a family of five children there are more boys than girls.

$(\boldsymbol{Answer};\ \mathbf{0.50})$

92. X is $B(\ 4, p)$ and $p(X = 4) = 0.0256$. find $p(X = 2)$

$(\boldsymbol{Answer};\ \mathbf{0.3456})$

93. X~ $B(\ n, 0.3)$. find the least possible value of n such that $P(\ X\ \geq 1) = 0.8$

$(\boldsymbol{Answer};\ \mathbf{5}\ )$

94. The variable X is $B(\ n, 0.6)$ and $P(\ X\ < 1) = 0.0256$. Find the value of n.

$(\boldsymbol{Answer};\ \mathbf{4}\ )$

95. X is $B(\ n, p)$ with mean 3 and standard deviation 2. Find the values of n and p.

$(\boldsymbol{Answer};\ \boldsymbol{p} = \mathbf{0.2}, \boldsymbol{n} = \mathbf{25}\ )$

96. The random variable X is $B(10, p)$ where $p < 0.5$. The variance of X is 1.875. find

a) The value of p

b) $E(X)$

c) $P(X = 2)$

$$(\boldsymbol{Answer}; \ \boldsymbol{a})\ \mathbf{0.25}, \quad \boldsymbol{b})\ \mathbf{2.5} \quad \boldsymbol{c}) \quad \mathbf{0.282}\ )$$

97. In a certain African village, 80% of the villagers are known to have a particular eye disorder. Twelve people are waiting to see the nurse.

a) What is the most likely number to the disorder?

b) Find the probability that fewer than half have the eye disorder.

$$(\boldsymbol{Answer}; \ \boldsymbol{a})\ \mathbf{10}, \ \boldsymbol{b})\ \mathbf{0.00039}\ )$$

98. In a bag, there are six red counters, eight yellow counters and six green counters. An experiment consists of taking a counter at random from the bag, noting its color and then replacing it in the bag. This procedure is carried out ten times in all. Find

a) The expected number of red counters drawn,

b) The most likely number of green counters drawn,

c) The probability that no more than four yellow counters are drawn.

$$(\boldsymbol{Answer}; \ \boldsymbol{a})\ \mathbf{3}, \quad \boldsymbol{b})\ \mathbf{3} \quad \boldsymbol{c}) \quad \mathbf{0.633}\ )$$

99. The random variable X is distributed binomially with mean 2 and variance 1.6. find

a) The probability that X is less than 6,

b) The most likely value of X.

$(\boldsymbol{Answer}; \ \boldsymbol{a})\ \mathbf{0.994}, \ \boldsymbol{b}) \quad \mathbf{2}\ )$

100. A continuous random variable has p.d.f $f(x) = kx^2$ for $0 \leq x \leq 4$.

a) Find the value of the constant k

b) find $P1 \leq X \leq 3)$

$$\left(\boldsymbol{Answer};\ \boldsymbol{a})\ \boldsymbol{k} = \frac{\mathbf{3}}{\mathbf{64}},\ \ \boldsymbol{b})\ \mathbf{0.4063} \right)$$

101. the continuous random variable X has a p.d.f f(x) where

$$f(x) = \begin{cases} k(x+2)^2, & -2 \leq x \leq 0 \\ 4k, & 0 \leq x \leq 1\frac{1}{3} \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of the constant k

b) Sketch $y = f(x)$

c) Find $P(-1 \leq X \leq 1)$

d) Find $P(X > 1)$

$$\left(\boldsymbol{Answer};\ \boldsymbol{a})\ \boldsymbol{k} = \frac{\mathbf{1}}{\mathbf{8}},\ \ \boldsymbol{c})\ \frac{\mathbf{19}}{\mathbf{24}},\ \ \boldsymbol{d})\ \frac{\mathbf{1}}{\mathbf{6}} \right)$$

102. Given a continuous random variable X such that;

$$f(x) = \begin{cases} 4x(1 - x^2), & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that X is a random variable. I.e. a p.d.f

b) Find the cumulative distribution function of X

c) Find the median

d) Sketch the cumulative distribution function.

103. the continuous random variable X has p.d.f f(x) where $f(x) = k(x+2)^2, 0 \leq x \leq 2$

a) find the value of the constant k

b) find $P(0 \leq X \leq 1)$ and hence find $P(X > 1)$

$$\left(\textbf{\textit{Answer}};\ \ \boldsymbol{a})\ \boldsymbol{k} = \frac{\mathbf{3}}{\mathbf{65}},\ \ \boldsymbol{b})\ \frac{\mathbf{19}}{\mathbf{56}},\frac{\mathbf{37}}{\mathbf{56}} \right)$$

104. the continuous random variable X has a p.d.f f(x) where

$$f(x) = \begin{cases} k, & 0 \le x < 2 \\ k(2x-3), & 2 \le x \le 3 \\ 0 \text{ otherwise} \end{cases}$$

a) find the value of the constant k

b) sketch $y = f(x)$

c) find $P(X \le 1)$

d) find $P(X > 2.5)$

e) find $P(1 \le X \le 2.3$

$$\left(\textbf{\textit{Answer}};\ \ \boldsymbol{a})\ \boldsymbol{k} = \frac{\mathbf{1}}{\mathbf{4}},\ \ \boldsymbol{c})\ \mathbf{0.25},\quad \boldsymbol{d})\ \mathbf{0.3125},\qquad \boldsymbol{e})\ \mathbf{0.3475} \right)$$

105. the continuous random variable X has a p.d.f f(x) where

$$f(x) = \begin{cases} kx, & 0 \le x < 1 \\ k, & 1 \le x < 3 \\ k(4-x), & 3 \le x \le 4 \\ 0 \text{ otherwise} \end{cases}$$

a) Draw a sketch for $y = f(x)$

b) Find k

c) Find E(X)

$$\left(\textbf{\textit{Answer}};\ \ \boldsymbol{b})\ \boldsymbol{k} = \frac{\mathbf{1}}{\mathbf{3}},\ \ \boldsymbol{c})\ \mathbf{2} \right)$$

106. A random variable X has a probability density function f given by

$$f(x) = \begin{cases} cx(5-x), & 0 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$$

Show that $c = \frac{6}{125}$ and find the mean X

$$(\textbf{\textit{Answer}};\ \ \mathbf{2.5.})$$

107. The mass, X kg, of a particular substance produced in an hour in a chemical process is modelled by a continuous random variable with probability density function given by,

$$f(x) = \frac{3}{32}x^2, \qquad 0 \le x < 2,$$
$$f(x) = \frac{3}{32}(6 - x), \quad 2 \le x \le 6$$
$$f(x) = 0, \quad \text{otherwise}$$

a) sketch a graph of f
b) find $P(X < 4)$
c) find the mean mass produced per hour
d) the substance is sold at ε100 per kilogram and the running cost of the process is ε20 per hour. Taking εY as the profit made in each hour, express Y in terms of X
e) find the expected value of Y

$$\left(\boldsymbol{Answer};\ \boldsymbol{b})\ \boldsymbol{k} = \frac{\mathbf{13}}{\mathbf{16}},\ \boldsymbol{c})\ 2\frac{7}{8} \quad \boldsymbol{d})\ \boldsymbol{Y} = \mathbf{100}\boldsymbol{X} - \mathbf{20},\ \boldsymbol{e})\ \mathbf{267}\frac{\mathbf{1}}{\mathbf{2}} \right)$$

108. a continuous random variable X has a p.d.f f(x) where

$$f(x) = \begin{cases} kx, & 0 \le x < 1 \\ k(2 - x), & 1 \le x \le 2 \\ 0 \text{ otherwise} \end{cases}$$

Find:

a) the value of the constant k
b) E(X)
c) Var(X)
d) $P(\frac{3}{4} \le X \le 1\frac{1}{2})$
e) The mode

$$\left(\boldsymbol{Answer};\ \boldsymbol{a})\ \boldsymbol{k} = \mathbf{1},\ \boldsymbol{b})\ \mathbf{1}\ \ \boldsymbol{c})\ \frac{\mathbf{1}}{\mathbf{6}}\ \ \boldsymbol{d})\ \frac{\mathbf{19}}{\mathbf{32}}\ \ \boldsymbol{e})\ \mathbf{1} \right)$$

109. A continuous random variable X has a probability density function f given by

$$f(x) = \frac{k}{x(4-x)} \quad 1 \leq x \leq 3$$
$$f(x) = 0 \;\; \text{otherwise}$$

a) Show that $k = \frac{2}{\text{In } 3}$

b) Calculate the mean and variance of X

$$(\boldsymbol{Answer};\;\; \boldsymbol{b})\; \mathbf{2}, \qquad \mathbf{0.3590})$$

110. The random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ x^4, & 0 \leq x \leq 1 \\ 1 & x \leq 1 \end{cases}$$

Find:

a) $P(0.3 < X < 0.6)$

b) The median m

c) The value of a such that $P(X > a) = 0.4$

$$(\boldsymbol{Answer};\;\; \boldsymbol{a})\;\; \mathbf{0.1215},\;\; \boldsymbol{b})\; \mathbf{0.841}\;\; \boldsymbol{c})\; \mathbf{0.880}\;)$$

111. The continuous random variable X has continuous p.d.f f(x) where

$$f(x) = \begin{cases} \frac{x}{3} - \frac{2}{3}, & 2 \leq x \leq 3 \\ \alpha, & 3 \leq x \leq 5 \\ 2 - \beta x, & 5 \leq x \leq 6 \\ 0 \text{ otherwise} & \end{cases}$$

Find;

(a) α and β,

(b) $F(x)$ and sketch $y = F(x)$

(c) $P(2 \leq X \leq 3.5)$,

(d) $P(X \geq 5.5)$

$$\left(\boldsymbol{Answer};\ \boldsymbol{a})\ \frac{\mathbf{1}}{\mathbf{3}}, \frac{\mathbf{1}}{\mathbf{3}}\quad \boldsymbol{c})\ \frac{\mathbf{1}}{\mathbf{3}}\ \ \boldsymbol{d})\ \frac{\mathbf{1}}{\mathbf{24}} \right)$$

112. A continuous random variable X has a c.d.f given as

$$P(X \leq x) = \begin{cases} 2\beta(x-1)^2 & , \quad 1 \leq x < 3 \\ \beta(14x - x^2 - 25), & \quad 3 \leq x < 7 \\ 1 & , x \geq 7 \end{cases}$$

Determine the

(a) Value of β,

(b) Median of x

(c) Pdf of x. sketch the pdf

113. Given the continuous random variable, X has a pdf f(x) as

$$f(x) = \begin{cases} tx, & 0 \leq x < 2 \\ t(4-x), & 0 \leq x < 4 \\ 0 & , \text{elsewhere} \end{cases}$$

(a) Find the pdf.

(b) Determine the mode of x

(c) Calculate the standard deviation of x.

114. The random variable X is B (6, 0.42). Find

a) $P(X=5)$, b) $P(x=3)$ c) $P(x \leq 2)$

115. An unbiased die is thrown seven times. Find the probability of throwing at least 5 fives.

116. A fair coin is tossed six times. Find the probability of throwing at least four tails.

117. X is B (4,ρ) and P(X=4) =0.0256. find P(X=2)

118. A coin is biased so that it is twice as likely to show heads as tails. The coin is tossed five times. calculate the probability that

a) Exactly three heads are obtained ,

b) More than three are obtained.

119. The probability that a target is hit is 0.3. Find the least number of shots, which should be fired if the probability that a target is hit at least once is greater than 0.85.

120. In a test, there are ten multiple-choice questions. For each question of five answers, only one of which is correct. A student guesses each of the answers.

a) Find the probability that he gets more than seven correct.

He needs to obtain over half marks to pass and each question carries equal weight.

b) Find the probability that he passes the test.

121. A continuous random variable has a p.d.f. $f(x) = 3x^k$ for $0 \leq x \leq 1$. Where k is a positive integer. Find

a) the value of the constant k.

b) The mean of X

c) The value of x such that $P(X \leq x) = 0.5$

(***Answer***; ***a***) **2** ***b***) **0.75** ***d***) **0.794**)

122. The continuous random variable X has a cumulative distributive function F(x) given by

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 2x - x^2 & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

a) Show that $P\left(X < \frac{1}{2}\right) = \frac{3}{4}$

b) Sketch $y = f(x)$

c) Find the inter quartile range of X

(***Answer***; ***c***) **0.3661**)

123. The continuous random variable X has a p.d.f. f given by

$$f(x) = \begin{cases} cx(5 - x) & 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Show that $c = \frac{6}{125}$ and find the mean of X.

(***Answer***; ***a***) ***b***) ***d***))

124. A random variable X has a probability density function

$$f(x) = Ax(6-x)^2 \quad 0 \le x \le 6$$
$$= 0 \qquad \text{elsewhere.}$$

a) Find the value of the constant A
b) Calculate
   i) The mean,
   ii) The mode
   iii) The variance
   iv) The standard deviation of X.

125. The monthly supply for petrol in thousands of units for a company is continuous random variable x with a probability density of the form;

$$f(x) = \begin{cases} ax^2(d-x); & 0 \le x \le 1 \\ 0 \qquad . & \text{elsewhere} \end{cases}$$

(a) Given that the average supply per month is 600 units, determine the value of a and d.
(b) Find P(0.9<x<1)

126. A continuous random variable X has p.d.f. given by $f(x) = \begin{cases} kx^2 & 0 \le x \le 1 \\ k(2-x) & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

Find

a) The value of the constant k,
b) E(X)
c) Var (X)
d) $P\left(\frac{3}{4} \le X \le 1\frac{1}{2}\right)$,

e) The mode.

127. The random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ x^4 & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

Find:

a) $P(0.3 < X < 0.6)$,

b) The median m

c) The value of a such that $P(X > a) = 0$.

128. The cumulative distribution function F(x0 is defined by

$$F(x) = \begin{cases} 2x - 2x^2; & 0 \leq x \leq 0.25 \\ a + x; & 0.25 \leq x \leq 0.5 \\ 1 & x \geq 0.5 \end{cases}$$

i. Find the values of a and b

ii. Sketch the p.d.f of f(x) and hence find $p(x \leq 0.35/x > 0.1)$

129. The continuous random variable X has (cumulative) distribution function given by

$$F(x) = \begin{cases} \frac{1+x}{8} & -1 \le x \le 0 \\ \frac{1+3x}{8} & 0 \le x \le 2 \\ \frac{5+x}{8} & 2 \le x \le 3 \end{cases}$$

Where $F(x) = 0$ for $x < -1$, and $F(x) = 1$ for $x > 3$.

a) Sketch the graph of the p.d.f. $f(x)$.

b) Determine the expectation of X and the variance of X.

c) Determine $P(3 \le 2X \le 5)$.

$$\left(\boldsymbol{Answer}; \ \boldsymbol{b)} \ \mathbf{1}, \frac{\mathbf{5}}{\mathbf{6}} \quad \boldsymbol{c)} \ \mathbf{0.25} \right)$$

130. The random variable X has probability density function

$$f(x) = \begin{cases} 4x^k & 0 \le x \le 1 \\ 0 & \text{Otherwise} \end{cases}$$

Where k is a positive integer.

Find

a) The value of k

b) The mean of X

c) The value , x such that $P(X \le x) = 0.5$.

131. The continuous random variable X has a probability density function f(x) where

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 3 \\ a(4-x) & 3 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

a) Sketch the function f(x), hence, the value a

b) Determine cumulative probability function F(X). Hence, find the median of X.

c) Sketch F(X).

132. On any day , the amount of time ,measured in hours, that Mr. Google spends watching television is a continuous random variable T, with cumulative distribution function given by

$$F(t) = \begin{cases} 0 & t \le 0 \\ 1 - k(15-t)^2 & 0 \le t \le 15 \\ 1 & t \ge 15 \end{cases}$$

Where k is a constant

a) Show that $k = \frac{1}{225}$ and find $P(5 \le T \le 10)$

b) Show that ,for $0 \le t \le 15$, the probability density function of T is given by

$f(t) = \frac{2}{15} - \frac{2}{225}t.$

c) Find the median of T.

(***Answer***; ***c***) **4.393**)

133. The continuous random variable X has probability density function f(x) defined by

$$f(x) = \begin{cases} \dfrac{c}{x^4} & x < -1 \\ c(2 - x^2) & -1 \le x \le 1 \\ \dfrac{c}{x^4} & x > 1 \end{cases}$$

a) Show that $c = \frac{1}{4}$.

b) Determine the cumulative distribution function F(x).

c) Determine the expected value of X and the variance of X.

$$\left(\boldsymbol{Answer};\ \boldsymbol{c})\ \mathbf{0}, \frac{\mathbf{11}}{\mathbf{15}} \right)$$

134. Given the cumulative distribution function F(X) as follow

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{12}(2 + x) & -2 \le x \le 0 \\ \frac{1}{6}(1 + x) & 0 \le x < 4 \\ \frac{1}{12}(6 + x) & 4 \le x < 6 \\ 1 & x \ge 6 \end{cases}$$

a) Sketch F(x).

b) Find the p.d.f of X and hence sketch it.

c) Find the mean and median.

d) Find the semi interquartile range.

$$(\boldsymbol{Answer};\ \boldsymbol{c})\ \mathbf{2},\quad \boldsymbol{d})\qquad)$$

135. A continuous variable X is distributed at random between the values x = 0 and x = 2 and has a probability density function of $\mathbf{a}x^2 + \mathbf{b}x$. The mean is 1.25.

a) Show that $\mathbf{b} = \frac{3}{4}$, and the value of **a**

b) Find the variance of X.

c) Verify that the median value of X is approximately 1.3

d) Find the mode

$$(\,\boldsymbol{Answer};\ \boldsymbol{a})\ -\mathbf{0.1875},\quad \boldsymbol{b})\ \mathbf{0.2375}\ \ \boldsymbol{d})\ \mathbf{2}\,)$$

136. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} cx^2 & 0 \le x \le 2 \\ 2c(4-x) & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $c = 0.15$
b) Find the mean of X.
c) Find the lower quartile of X.
d) Find the probability that a single observation of X lies between the lower quartile and the mean.

$$(\,\boldsymbol{Answer};\ \boldsymbol{b})\ \mathbf{2.2}\quad \boldsymbol{b})\ \mathbf{1.71},\qquad \boldsymbol{d})\ \mathbf{0.264}\,)$$

137. A random variable X has a cumulative distribution function give as

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x \le 1 \\ (\,ax + b) & 1 \le x < 2 \\ \frac{1}{4}(5-x)(x-1) & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

Where a and b are constant; find;

(i) The values of a and b

(ii) Hence from F(x), find $P(\,1.5\ \le x\ \le 2.5\,)$

(iii) The probability density function

(iv) $E(X)$ *and* $var(\,X)$

(v) Sketch $f(X)$ and F(x).

$(\boldsymbol{Answer};\ \boldsymbol{a})\quad \boldsymbol{b})\quad \boldsymbol{d})\quad)$

138. X has p.d.f defined by $f(x) = \frac{3}{80}(2 + x)(4 - x)$, for $0 \leq x \leq 4$. Find the mode and median of X

$(\boldsymbol{Answer};\ \mathbf{1},\ \dots\dots\dots\dots\dots)$

139. The marks obtained by the students are uniformly distributed with between 35% and 82%.

a) Write down the probability distribution of the marks (X)

b) The mean and variance of the distribution

c) Find F(x) hence use it to find $p(50 < X < 75)$

d) Find (i) $p(X \geq 48)$ and (ii) $p(X \leq 66)$.

$(\boldsymbol{Answer};\ \boldsymbol{a})\quad \boldsymbol{b})\quad \boldsymbol{d})\quad)$

140. The marks obtained by the students from the mock paper are uniformly distributed with between 40% and 69%.

e) Write down the probability distribution of the marks (X)

f) The mean and variance of the distribution

g) Find F(x) hence use it to find $p(51 < X < 63)$

h) Find (i) $p(X \geq 64)$ and (ii) $p(X \leq 59)$.

$(\boldsymbol{Answer};\ \boldsymbol{a})\quad \boldsymbol{b})\quad \boldsymbol{d})\quad)$

141. The continuous random variable Y has a rectangular distribution

$$f(y) = \begin{cases} \frac{1}{\pi}, & -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \\ 0 & \text{Otherwise} \end{cases}$$

a) Prove f(y) is a probability density function.

b) Find the mean and variance of Y

c) Sketch f(y)

d) Find $p(Y > 0)$

e) Find F(Y)

(Answers; b) $\mathbf{0}$, $\frac{\pi^2}{12}$ c) $\mathbf{0.5}$)

142. $X \sim N(-8, 12)$. Find

a) $P(X < -9.8)$,

b) $P(X > -8.2)$,

c) $P(-7 < X < 0.5)$

(Answers; a) $\mathbf{0.30153}$, $\boldsymbol{b})\ \mathbf{0.523125}$ $\boldsymbol{c})\ \mathbf{0.38602}$)

143. A random variable X is such that $X \sim N(-5,9)$

a) Find the probability that a randomly chosen item from the population will have a positive value

b) Find the probability that out of ten items chosen at random , exactly four will have a positive value.

(Answers; a) $\mathbf{0.04776}$, $\boldsymbol{b})\ \mathbf{0.0081}$)

144. $X \sim N(100,81)$. Find

a) $P(|X - 100| < 18)$,

b) $P(|X - 100| > 5)$,

c) $P(12 < X - 100 < 15)$

(Answers; a) $\mathbf{0.9545}$, $\boldsymbol{b})\ \mathbf{0.57822}$ $\boldsymbol{c})\ \mathbf{0.86097}$)

145. The life of a certain make of electric light bulb is known to be normally distributed with a mean life of 2000 hours and a standard deviation of 120 hours. Estimate the probability that the life of such a bulb will be

a) Greater than 2150 hours ,
b) Greater than 1910 hours
c) Within the range 1850 hours to 2090 hours

(Answers; a) $\mathbf{0.10565}$, $\boldsymbol{b})\ \mathbf{0.77337}$ $\boldsymbol{c})\ \mathbf{0.66772}$)

146. $X \sim N(100, \delta^2)$ and $p(X < 106) = 0.8849$.

a) Find the value of the standard deviation of δ.
b) Find the probability of obtaining more than the a average
c) Find the proportion of having less than 95.

(Answers; a) $\mathbf{50}$, $\boldsymbol{b})\ \mathbf{0.50}$ $\boldsymbol{c})\ \mathbf{0.460172}$)

147. A dog can jump and clear a height of 1.8m four in five attempts and a height of 1.45m one time out of ten attempts. Given that the height cleared are normally distributed,

a) Determine the mean (μ) and standard deviation (δ) of the height.
b) The percentage of cats clearing the height greater than 1.7m
c) Assuming the number of cats that follow distribution are 14, calculate the expected number of cats that can fail to clear 1.70m

(Answers; a) $\mathbf{1.6521}, \mathbf{0.1756}$, $\boldsymbol{b})\ \mathbf{0.39243}$ $\boldsymbol{c})\ \mathbf{6}$)

148. The probability that a dog can jump and clear a height of 1.9m is 0.75 and a height of 1.56m is 0.2. Given that the height cleared are normally distributed,

d) Determine the mean (μ) and standard deviation (δ) of the height.

e) The percentage of cats clearing the height greater than 1.3m

f) Assuming the number of cats that follow distribution are 15, calculate the expected number of cats that can fail to clear 1.65m

(Answers; a) $\mathbf{1.7488}$, $\mathbf{0.2243}$, $\boldsymbol{b})\ \mathbf{97.730}\%$ $\boldsymbol{c})\ \mathbf{3}$ **)**

149. A cat can jump and clear a height of 1.50m three in ten attempts and a height of 1.68m nine times out of ten attempts. Given that the height clears are normally distributed,

g) Determine the mean (μ) and standard deviation (δ) of the height.

h) The percentage of cats clearing the height greater than 1.7m

i) Assuming the number of cats that follow distribution are 16, calculate the expected number of cats that can fail to clear 1.73m

(Answers; a) 1.5522, 0.0997 , $\boldsymbol{b})\ \mathbf{6.917}\%$ $\boldsymbol{c})\ \mathbf{0}$ **)**

150. A dog can jump and clear a height of 1.8m four in five attempts and a height of 1.45m eight times out of ten attempts. Given that the height cleared are normally distributed,

j) Determine the mean (μ) and standard deviation (δ) of the height.

k) The percentage of cats clearing the height greater than 1.7m

l) Assuming the number of cats that follow distribution are 14, calculate the expected number of cats that can fail to clear 1.70m

(Answers; a) , $\boldsymbol{b})$ $\boldsymbol{c})$ **)**

151. a) The random variable Y is such that Y~N(8,25). Show that , correct to three decimal places, $P(|Y - 8| < 6.2) = 0.890$.

b). Three random observations of Y are made. Find the probability that exactly two observations will lie in the interval defined by $P(|\ \mathrm{Y}-8\ |) < 6.2$.

(Answers; $\boldsymbol{b})\ \mathbf{0.261393}$

152. The marks of 500 candidates in an examination are normally distributed with a mean of 45 marks and a standard deviation of 20 marks
    a) Given that the pass mark is 41 , estimate the number of candidates who passed the examination
    b) If 5% of the candidates obtain a distinction by scoring x marks or more , estimate the value of x.
    c) Estimate the interquartile range of the distribution

(Answers; a) $\mathbf{290},\ \boldsymbol{b})\quad \boldsymbol{c})\ \mathbf{)}$

153. $X \sim N(400,64)$.
    a) Find the limits within which the central 95% of the distribution lies.
    b) Find the interquartile range of the distribution.
154. The random variable X is distributed $N(\mu, 12)$ and it is known that $P(X > 32) = 0.8438$. find the value ofμ.
155. The random variable X is distributed $N(\mu, \sigma^2)$. $P(X > 80) = 0.0113$ and $P(X < 30) = 0.0287$.find μ and σ
156. The random variable X is $N(\mu, \sigma^2)$. $P(X < 35) = 0.2$ and $P(35 < X < 45) = 0.65$. find μ and σ. Hence, determine percentage of obtaining more than 50%.
157. The random variable X is $N(\mu, \sigma^2)$. $P(X < 40) = 0.3$ and $P(40 < X < 70) = 0.70$. find μ and σ. Hence, determine percentage of obtaining less than 80%.
158. The masses of sugar are normally distributed. In a large consignment of packets of sugar, it is found that 5% of them have greater than 510g and 2% have masses greater than 515g. Estimate the mean and standard deviation of the distribution.
159. The marks in an examination were found to be normally distributed. 10% of the candidates were awarded a distinction for obtaining over 75.

20% of the candidates failed the examination with a mark of under 40. Find the mean and standard deviation of the distribution of marks.

160. A farmer cuts hazel twigs to make into beanpoles to sell at the market. He says that a stick is 240 cm long. In fact, the lengths of the sticks are normally distributed and 55% are over 240 cm long. 10% are over 250 cm long.
Find the probability that a randomly selected stick is shorter than 235 cm

161. Tea is sold in packages marked 750 g. the masses of the packages are normally distributed with a mean of 760 g. it is known that less than 1% of packages are under weight. What is the maximum value of the standard deviation of distribution?

162. If $X \sim N(80, 36)$ and $p(|X - 80| < C) = 0.9$

i) Value of C

ii) The limit within which the central 90% of the distribution lies.

163. Given that X is a continuous random variable which is normally distributed with mean (μ) standard deviation (δ) such that $p(X < 30) = 0.4$ and $p(30 < X < 50) = 0.3$, find the;

a) value of μ and δ

b) Semi inter quartile range.

164. Given that X is a continuous random variable which is normally distributed with mean (μ) standard deviation (δ) such that $p(X < 40) = 0.35$ and $p(40 < X < 70) = 0.4$, find the;

a) value of μ and δ

b) Inter quartile range.

165. Machine components are mass-produced at a factory. A customer requires that the components should be 5.2 cm long but they will be acceptable if they are within limits 5.195 cm to 5.205 cm. the customer tests the components and finds that 10.75% of those supplied are over-size and 4.95% are under-size. Find the mean and standard deviation of the lengths of the components supplied, assuming that they are normally distributed.
If three of the components are selected at random what is the probability that one is under-size, one is over-size and one is satisfactory?

166. A machine dispenses peanuts into bags so that the weight of peanuts in a bag is normally distributed.

a) Initially the mean weight of peanuts in a bag is 128.5 g and the standard deviation is 1.5 g. find the probability that the weight of peanuts I a randomly chosen bag exceeds 130 g.

b) The machine is given a minor overhaul that changes the mean weight, μ, of peanuts in bag without affecting the standard deviation. Following the overhaul, 14% of bags contain more than 130 g of peanuts. Find, to 4 significant figures, the new value of μ.

c) Later the machine requires a major repair, following which the mean weight of peanuts in a bag is 128.3 g and 4% of bags contain less than 126 g. find, to 3 s.f.g., the standard deviation of the weight of peanuts in a bag after this major repair.

167. Apples imported to Uganda are in batches of 1000. The probability that a batch contains a spoilt apple is 0.025. What is the probability that the batch has;

i. Exactly 20 spoilt apples.

ii. At least 27 spoilt apples.

iii. Less than 18 spoilt apples.

168. The masses in grams of 13 washers selected from a production line at random are; 15.4, 15.2, 14.6, 16.1, 14.8, 15.3, 15.9, 16.0, 15.4, 14.6, 15.0, 15.5, 16.1.

Calculate the 95% and 96%confidence limits for the mean mass of the washers on this particular production lines, assuming that the masses can be modelled by a normal distribution.

169. The masses in grams of 13 washers selected from a production line at random are; 5.4, 5.2, 4.6, 6.1, 4.8, 5.3, 5.9, 6.0, 5.4, 4.4, 5.0, 5.5, 6.6.

Calculate the 99% and 92%confidence limits for the mean mass of the washers on this particular production lines, assuming that the masses can be modelled by a normal distribution.

170. The height, $x\ cm$ each student in the random sample of 200 students living in Wakiso district was measured. The following results were obtained

$\sum X = 35{,}000,\ and\ \sum X^2 = 6{,}200{,}000.$

a) Calculate the unbiased estimate of the ean and the variance of the height of students living in Wakiso district.

b) Determine the 90% and 76% confidence interval for the height of men living in Wakiso district.

$$\textit{Answer } a)\ \mathbf{175, 376.884}\ \ b)\ \ \mathbf{172.742}\ \le\ \boldsymbol{\mu} \le \mathbf{177.258}\,,$$

171. A random sample of 100 observations from a normal population with mean μ gave the following data; $\sum X = 82{,}00,\ \ and\ \ \sum X^2 = 686{,}000$.

a) Find a 98% confidence interval for μ.

b) Find a 99% confidence limit for μ.

172. A random sample of 100 taxis inspected on a road on a particular day gave the ages, X, summarized below: $\sum X = 560, \sum X^2 = 3286$. determine the:

i. Unbiased estimate for the variance of all the taxis on the road.

ii. 91.869% Confidence limits for the mean age of all the taxis that operate on the road.

iii. 56% Confidence interval for the mean age of all the taxis on the road.

131. A random sample of 110 taxis inspected on a road on a particular day gave the ages, X, summarized below: $\sum fX = 580, \sum fX^2 = 44024$. determine the:

a) Estimate 95% confidence limits for the mean age of all the taxis that operate on the road.

b) Estimate 56% confidence interval for the mean age of all the taxis on the road.

173. A sample of 81 mangoes is taken from a large consignment of mangoes and their masses in grams, $x_{1,}x_2, x_3, \dots \dots \dots .., x_n$ such that$\sum_{i=1}^{81} x_i = 176.70$, and $\sum_{i=1}^{81} x_i$ =424.51.

i. Calculate the unbiased estimate for the variance of the masses of mangoes in the consignment.

ii. Determine the 98% confidence interval for the mean mass of mangoes in the consignment

NUMERICAL METHODS.

1. The sides of a rectangle are measured as 4.00m and 7.3m, to the nearest meters.

(a) Calculate the maximum possible error in each side

(b) Calculate the maximum and minimum values of the perimeter. Hence, determine the absolute error in the perimeter.

(c) Calculate the error in the area.

(Answers; a) $\mathbf{0.005, 0.05,}$ $\boldsymbol{b}$) $\mathbf{22.710, 22.49, 0.11}$ $\boldsymbol{c}$) $\mathbf{0.2451}$)

2. The sides of a rectangle are measured as 10.1m and 2.34m, to the nearest meters.

a) Calculate the maximum possible error in each side

b) Calculate the maximum and minimum values of the perimeter. Hence, determine the absolute error in the perimeter.

c) Calculate the error in the area.

d) Find the percentage error in the perimeter.

(Answers; a) $\mathbf{0.05, 0.005}$ $\boldsymbol{b}$) $\mathbf{24.990, 24.770, \ 0.11}$ $\boldsymbol{c}$) $\mathbf{0.1675}$ $\boldsymbol{d}$) $\mathbf{0.44\%}$)

3. If $x = 2.34$ and $y = 7.2$, both numbers are rounded off to the given number of decimal places. Find the maximum and minimum value of;

i) $y - x$,

ii) $2y + x$,

iii) $\frac{x}{y}$

iv) $\frac{x-y}{x+y}$. Hence, determine the absolute error in each expression.

$\boldsymbol{Answer\ i)\ 4.950, 4.805, 0.055\ ii)\ 16.845, 16.635, 0.105\ iii)\ 0.3280, 0.3221, 0.003}$

$$iv) -0.5008, -0.5182, 0.0087$$

4. Given that $p = \frac{12.4}{2.2} - \frac{10.80}{5.124}$,

i) Determine the range of values within which p lies. Hence, obtain the absolute error in p

ii) Determine the percentage error made in approximating p.

$$\textbf{Answer } i)\ 3.380 \le P \le 3.6842, \quad 0.1521, \quad ii)\ 4.3105\%$$

5. Give that $x = 4.96$ and $y = 2.013$ each rounded off to the given number of decimal places. Calculate the maximum possible error in $\frac{x}{x+y}$.

$$\textbf{Answer } 0.0013$$

6. Obtain the interval within which the exact value of $3.24 + \frac{1.002 * 1.23}{0.9876}$ lies. Hence, obtain the percentage error in made in approximating the value.

$$\textbf{Answer } i)\ [4.4772, \quad 4.4987] \quad 0.2395\%$$

7. Given that $y = -4.2$, rounded off to the given number of decimal places. Find;

i) Limits within which y lies,

ii) The percentage error in y.

$$\textbf{Answer } i)\ \textbf{lower limit} = -4.1550, \textbf{upper limit} = -4.25 \quad ii)\ 1.1905\%$$

8. Given that $p = -3.41$, rounded off to the given number of decimal places. Find;

iii) Limits within which p lies,

iv) The relative error in p.

$$\textbf{Answer } i)\ \textbf{lower limit} = -3.405, \textbf{upper limit} = -3.4150 \quad ii)\ 0.0015\%$$

9. The value of $l = 100.23$m was obtained when measuring the length of the football pitch. Given that, the percentage error in this value was 0.08%. Find the range within which the value of l lies.

$$\textit{Answer } i)\ \mathbf{100.1498} \le \boldsymbol{l} \le \mathbf{100.3102}$$

10. The value of $q = 20.78$m was obtained when measuring the length of the square. Given that, the percentage error in this value was 0.06%. Find the limits within which the value of q lies. Hence, determine the percentage error in the area of the square.

$$\textit{Answer } i)\ \textit{lower limit} = \mathbf{20.7925},\ \textit{upper limit} = \mathbf{19.9875}\ \ ii)\ \ \mathbf{0.0012}\%$$

11. Given numbers $x = 2.24$ and $y = 4.8$, with percentage errors 0.4% and 0.6% respectively. Find the percentage error inx^3y.

$$\textit{Answer}\ \ \mathbf{1.80}\%$$

12. Given numbers $p = 10.0$ and $q = 12.11$, with percentage errors 0.4% and 0.6% respectively. Find the percentage error in $p + 3q$.

$$\textit{Answer}\ \ \mathbf{0.5568}\%$$

13. The numbers $A = 12.4$, $B = 29.444$ and $C = 2.25$ are each rounded off with percentage errors 2.5%, 0.05% and 1% respectively. Find the;

(i) limits within which the exact value of $\frac{A}{(B-C)^2}$ lies'

(iii) Percentage error made in $\frac{A}{(B-C)^2}$

$$\textit{Answer } i)\ \textit{lower limit} = \mathbf{0.0172},\ \textit{upper limit} = \mathbf{0.0163}\ \ ii)\ \ \mathbf{2.6837}\%$$

14. A trader in phones and computers makes annual profits in phones of sh. 800 million with a margin of error of sh. 58 million and annual loss in computer of sh. 200 million with a margin error of sh. 5 million.

a) Find the limits of values corresponding to her gross income.

b) Find the percentage error in the gross income to two decimal places.

$$\boldsymbol{Answer\ i)\ lower\ limit = 537, upper\ limit = 663\ \ ii)\ \ 10.50\%}$$

15. A trader in books and pens makes annual profits in books of sh. 60 million with a margin of error of sh. 600000 and annual profits in pens of sh. 20 million with a margin error of sh. 350000.

i) Find the range of values corresponding to her gross income.

ii) Find the relative error in the gross income to three decimal places.

$$\boldsymbol{Answer\ i)\ \ 79.50 \leq income \leq 80.950\ \ ii)\ \ 0.012\%}$$

16. A trader in phones and computers makes annual profits in phones of sh. 800 million with a margin of error of 2.5% and annual loss in computer of sh. 200 million with a margin error of 5%.

c) Find the interval of values corresponding to her gross income.

d) Find the percentage error in the gross income.

$$\boldsymbol{Answer\ i)\ \ 570 \leq income \leq 630\ \ ii)\ \ 5.0\%}$$

17. A company had a capital of sh. 500 million. The profit in a certain year was sh. 25.8 million in section A of the company and sh. 14.56 million in section B of the company. There was possible error of 5% in section A and an 8% error in section B.

i) Find the maximum and minimum values of the total profit of the sections as a percentage of the capital.

ii) Find the limits with in which the exact profits from the both companies lies.

$$\boldsymbol{Answer\ i)\ \ 8.5630, 7.581\ \ ii)\ \ lower\ limit = 37.9052, upper\ limit = 42.8148}$$

18. A company dealing in clothes and shoes had a capital of sh. 700 million. The profit in a certain year was sh. 50 million from clothes and sh. 43.6 million from shoes. There was possible error of 5.5% from the clothes and 11% error from shoes.
    a) Find the maximum and minimum values of the total profit of the departments as a percentage of the capital.
    b) Find the limits with in which the exact profits from the both departments companies lies.

$$\boldsymbol{Answer\ i)\ \ 14.4494, 12.2934\ \ ii)\ \ lower\ limit = 86.054, upper\ limit = 101.146}$$

19. The radius of the circle is measured as 4.40m to the nearest cm. calculate the upper bound of its area correct to four significant figures. Hence, the percentage error in approximating the area.

$$\boldsymbol{Answer\ \ 60.96, \qquad 0.2302\%}$$

20. The radius of the circle is measured as 2.423m to the nearest cm. calculate the lower bound of its area correct to three decimal places. Hence, the percentage error in approximating the area.

$$\boldsymbol{Answer\ \ 18.437, \qquad 0.041\%}$$

21. The cylindrical pipe has a radius of 2.5m measured to the nearest units. If the relative absolute error made in calculating its volume is 0.125. Find the absolute relative error made in measuring its height. (**Answer: 0.085**)

22. The cylindrical pipe has a radius of 4.65m measured to the nearest units. If the relative absolute error made in calculating its volume is 0.025. Find the absolute relative error made in measuring its height. (**Answer: 0.0228**)

23. The cylindrical pipe has a height of 2.6m measured to the nearest units. If the relative absolute error made in calculating its volume is 0.216. Find the absolute relative error made in measuring its radius. (**Answer: 0.0984**)

24. The cylindrical pipe has a height of 7.76m measured to the nearest units. If the relative absolute error made in calculating its volume is 0.3. Find the absolute relative error made in measuring its radius. (**Answer: 0.1497**)

25. Two positive decimal numbers x and y are rounded to give X and Y with Δx and Δy respectively. Show that the maximum relative error made in approximating $x + y$ by $X + Y$ is given by

$$\frac{|\Delta x|+|\Delta y|}{X+Y}$$

26. Two positive decimal numbers x and y are rounded to give X and Y with Δx and Δy respectively. Show that the maximum relative error made in approximating $x - y$ by $X - Y$ is given by

$$\frac{|\Delta x|+|\Delta y|}{X-Y}$$

27. Two decimal numbers x and y are rounded to give X and Y with Δx and Δy respectively. Show that the maximum relative error made in approximating $\frac{x}{y}$ by $\frac{X}{Y}$ is given by

$$\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$$

28. Two decimal numbers x and y are rounded to give X and Y with Δx and Δy respectively. Show that the maximum relative error made in approximating xy by XY is given by

$$\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$$

29. Two decimal numbers x and y are rounded to give X and Y with Δx and Δy respectively. Show that the maximum relative error made in approximating x^2y by X^2Y is given by

$$2\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$$

30. Two decimal numbers X and Y are rounded to give x and y with Δx and Δy respectively. Show that the maximum relative error made in approximating X^2Y by x^2y is given by

$$2\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$$

31. Numbers X and Y were estimated with maximum possible errors of Δx and Δy respectively. Show that the maximum possible relative error is the estimation of xy^2 is given by: $\left|\frac{\Delta x}{X}\right|+2\left|\frac{\Delta y}{Y}\right|$.

32. Given that a=1.50, b=13.3 and c=9.200, are all rounded off to the given decimal places. Find the minimum value of;

i. $\frac{a+b}{c}$

ii. $\frac{a-c}{c^2}$

iii. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

(***Answer i*)1.6026** ***ii*) − 0.0910** ***iii*) 0.1843)**

33. Show that the maximum relative error in the $x\sqrt{y}$ as $X\sqrt{Y}$ is given by $\left[\left|\frac{\Delta x}{x}\right| + \frac{1}{2}\left|\frac{\Delta y}{y}\right|\right]$. Hence, deduce the relative error in $x^3\sqrt[4]{y}$ whereΔx and Δy are errors in approximating x andy respectively.

34. Given that $a = 2.5$, $b = 6.00$ are rounded off to the given number of decimal places, determine the interval within which the exact value of

(i) $\frac{b-a}{a+b}$,

(ii) $\frac{a-b}{b+a}$,

(iii) $\frac{a-b}{ab}$,

(iv) $\frac{a+b}{a-b}$ Lies. [Reference question UNEB 2010 Number 14b(ii)]

(Answers; i) ii) iii) iv))

35. Show that the maximum relative error in approximating $x\sqrt{y}$ as $X\sqrt{Y}$ is given by $\left[\left|\frac{\Delta x}{X}\right| + \frac{1}{2}\left|\frac{\Delta y}{Y}\right|\right]$. Hence, deduce the relative error in $x^3\sqrt[4]{y}$ whereΔx and Δy are errors in approximating x andy respectively.

36. Two positive decimal numbers x and y are rounded to give X and Y with Δx and Δy respectively. Show that the maximum relative error made in approximating $x^{\frac{1}{2}}y$ by $X^{\frac{1}{2}}Y$ is given by

$$\frac{1}{2}\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$$

37. Two positive decimal numbers x and y are rounded to give X and Y with Δx and Δy respectively. Show that the maximum relative error made in approximating $\sqrt[3]{x}\,y$ by $\sqrt[3]{X}\,Y$ is given by

$$\frac{1}{3}\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$$

38. Given that Y_1 and Y_2 are approximations to X_1 and X_2 with error E_1 and E_2 respectively. Show that the maximum possible relative error in

$$\left|\frac{E_1}{Y_1}\right| + \left|\frac{E_2}{Y_2}\right|$$

39. Given that Y and Z are measured with possible errors ΔY and ΔZ respectively. show that the relative error in the product YZ is $\frac{Z|\Delta Y| + Y|\Delta Z|}{YZ}$

40. (a) Show that the relative error in solving A^2B is $2\left|\frac{e_1}{a}\right| + \left|\frac{e_2}{b}\right|$ where e_1 and e_2 are errors in a and b respectively. Given that the error in c is e_3, deduce that the relative error in $(a+c)^2b$ is $2\left|\frac{e_1+e_3}{a+c}\right| + \left|\frac{e_2}{b}\right|$, if $a = 12.2, b = 4.600$ and $c = -9.54$ recorded to the respective decimal places, determine the absolute error in $(a+c)^2b$

(***Answer*** **1.3495**)

41. If $p = \sin x$, determine the expression for the absolute error and maximum relative error.

(***Answer*** $\Delta \boldsymbol{x} \cos \boldsymbol{x}$, $\Delta \boldsymbol{x} \cot \boldsymbol{x}$,)

42. If $q = \frac{\sin x}{\cos x}$, determine the expression for the absolute error and maximum relative error.

(***Answer*** ,)

43. If $y = \tan\theta$, find the interval in which y lies given that $\theta = 45^0 \pm 0.4^0$

(***Answer*** $[\mathbf{1.4933}, \mathbf{1.7463}]$)

44. Given that $y = x\sin x$, derive the expression for the maximum absolute percentage error in y hence, if $x = 30^0$, find the maximum absolute percentage error in y to three decimal place.

(***Answer*** **1.735**)

45. Given that the error in measuring an angle is 0.5^0. Find the maximum possible error in $\frac{\cos x}{\sin x}$, if $x = 40^0$.

$$(\boldsymbol{Answer}\ \ \mathbf{0.00786}\ )$$

46. Given that $y = x\cos x$, derive the expression for the maximum absolute percentage error in y hence, if $x = 30^0$, find the maximum absolute percentage error in y to three decimal place.

$$\left(\boldsymbol{Answer}\quad \left(\left|\Delta \boldsymbol{x}\boldsymbol{tan}\boldsymbol{x} + \left|\frac{\Delta \boldsymbol{x}}{\boldsymbol{x}}\right|\right| \times \mathbf{100\%}\right)\ ,\quad \mathbf{4.4617\%}\right)$$

47. Given that $A = xy\sin\theta$ where x and y. Deduce that the maximum possible relative error in A is given by $\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right| + |\Delta\theta|\cot\theta$. Where Δx, Δy and $\Delta\theta$ are small numbers compared to x, y and θ respectively. Find the percentage error made in the area if $x = 2.5\text{cm}, y = 3.4\text{cm}$ and $\theta = 30^0$

(**answer 1.769% 4.98**)

48. The maximum possible error in the values of $\sin x$ and $\cos x$ is± 0.00005. What are the corresponding maximum and minimum values of $\tan x$ for $x = 1.05$ radians? Give your answer correct to 3 decimal places).

$(\boldsymbol{Answer}\quad \mathbf{1.744, 1.743}\ )$

49. The height and radius of a cylinder are measured as h and r with maximum possible errors of Δh and Δr respectively. Show that the maximum possible error made in calculating the volume is $\left(\left|\frac{\Delta h}{h}\right| + 2\left|\frac{\Delta r}{r}\right|\right) \times 100.$

50. Determine the maximum absolute error in $\frac{\sqrt{z}}{x^2y^3}$, given that $x = 2.4, y = 5.4$ and $z = 1.8$ all numbers rounded off to the given number of decimal places.

(Answer 0.000123)

51. Determine the maximum absolute percentage error in $\frac{\sqrt[4]{z}}{x^4y^3}$, given that $x = 2.5, y = 5.4$ and $z = 3.89$ all numbers rounded off to the given number of decimal places. **(Answer 0.00002468)**

52. Determine the maximum absolute error in $\frac{\sqrt[3]{z}}{x^3y^6}$, given that $x = 4.0, y = 5.4$ and $z = 10.0$ all numbers rounded off to the given number of decimal places.

(Answer 0.000000128)

53. Determine the maximum absolute error in $\frac{mz}{x^2y^3}$, given that $x = 2.4, y = 5.4, m = 1.09$ and $z = 1.8$ all numbers rounded off to the given number of decimal places.

(Answer 0.000220)

54. By plotting graphs show that there is a positive root of the equation $x^3 = 3x + 3$.

(Answers; $\mathbf{2.11}$)

55. Show graphically that the equation $x = \ln(8 - x)$ has root between 1 and 2.

(Answers; $\mathbf{1.82}$)

56. Show graphically that the equation $x^3 = \ln(8 - x)$ has two roots.

(Answers; $\mathbf{1.38, -1.50}$)

57. Use graphical method to find a first approximation to the real root of $x^3 - 3x + 4 = 0$

(Answers; $\mathbf{-2.2}$)

58. Use graphical method to find a first approximation to the real root of $x^3 - 3x + 5 = 0$

(Answers; $\mathbf{-2.28}$)

59. Use graphical method to show that $x^3 - 3x - 3 = 0$ has root between -2 and -3.

(Answers; $\mathbf{2.1}$)

60. By plotting graphs, show that the equation $e^x + x - 4 = 0$ has only one real root.

(Answers; $\mathbf{1.07}$)

61. Given the equation $y = \sin x - \frac{x}{2}$, show by plotting suitable graphs on the same axes that the root lies between $\frac{\pi}{2}$ and $\frac{3\pi}{4}$.

(Answers; $\mathbf{1.895}$)

62. Show that $3x = 1 + \cos x$ has a root between 0.5 and 1.0.

(Answers; $\mathbf{0.607}$)

63. Show that $2x = 3 + \cos x$ has a root between 1.0 and 2.0

(Answers; $\mathbf{0.835}$)

64. Show graphically that the equation $x^3 + x^2 - x = 0$ has three different roots.

(Answers; $\mathbf{-1.618, 0, 0.618}$)

65. Show graphically that the equation $x^3 - 4x - 9 = 0$ has only one positive root.

(Answers; $\mathbf{2..707}$)

66. Find the value of $\sqrt[3]{30}$ to three decimal places.

(Answers; $\mathbf{3.107}$)

67. Find the value of $\sqrt[4]{45}$ to three decimal places.

(Answers; $\mathbf{2.590}$)

68. Approximate $\sqrt[3]{25}$ to two significant figures.

(Answers; $\mathbf{2.924}$)

69. Locate the three roots of the equation $x^3 + 5x^2 - 5 = 0$

(Answers; $\mathbf{-4.781, -1.138, \quad 0.919}$)

70. Locate the three roots of the equation $x^3 + 5x^2 - 18 = 0$

(Answers; $\mathbf{-3.640, -3.0, \quad 1.646}$)

71. Locate the three roots of the equation $x^3 + 5x^2 - 6 = 0$

(Answers; $\mathbf{-4.732, -1.268, \quad 1.0}$)

72. a) Show that the iterative formula for finding the fourth root of a number N, is given by $x_{n+1} = \frac{3}{4}\left(x_n + \frac{N}{3x_n^3}\right), n = 0, 1, 2, \ldots$

c) Draw a flow chart that;

i) Reads the number N and the initial approximation $x_{0,}$

ii) Computes and prints N and its fourth root to three decimal places.

c) Perform the dry for the flow chart taking $N = 40$ and $x_{0,} = 2.4$

(Answers; $\mathbf{2.515}$)

73. a) Show that the iterative formula for finding the fifth root of a number N, is given by $x_{n+1} = \frac{4}{5}\left(x_n + \frac{N}{4x_n^4}\right), n = 0, 1, 2, \ldots$

a) Draw a flow chart that;

- Reads the number N and the initial approximation $x_{0,}$
- Computes and prints the root to three decimal places.

c) Perform the dry for the flow chart taking $N = 50$ and $x_{0,} = 2.0$

(Answers; $\mathbf{2.187}$)

74. a) Show that the iterative formula for finding the tenth root of a number N, is given by $x_{n+1} = \frac{9}{10}\left(x_n + \frac{N}{9x_n^9}\right), n = 0, 1, 2, \ldots$

b) Draw a flow chart that;

i) Reads the number N and the initial approximation $x_{0,}$

ii) Computes and prints the root to three decimal places.

d) Perform the dry for the flow chart taking $N = 44$.

(Answers; $\mathbf{1.460}$)

75. a) Show that the iterative formula for finding the seventh root of a number N, is given by $x_{n+1} = \frac{6}{7}\left(x_n + \frac{N}{6x_n^6}\right), n = 0, 1, 2, \ldots$

a) Draw a flow chart that;

i) Reads the number N and the initial approximation $x_{0,}$

ii) Computes and prints the root to three decimal places.

b) Perform the dry for the flow chart taking $N = 54$.

(Answers; $\mathbf{1.768}$)

76. a) Show that the iterative formula for finding the seventh root of a number N, is given by $x_{n+1} = \frac{7}{8}\left(x_n + \frac{N}{7x_n^7}\right), n = 0, 1, 2, \ldots$

c) Draw a flow chart that;

i) Reads the number N and the initial approximation $x_{0,}$

ii) Computes and prints the root to three decimal places.

d) Perform the dry for the flow chart taking $N = 70$.

(Answers; $\mathbf{1.835}$)

77. a) Show that the iterative formula for finding the seventh root of a number N, is given by $x_{n+1} = \frac{6}{7}\left(x_n + \frac{N}{6x_n^6}\right), n = 0, 1, 2, \ldots$

a) Draw a flow chart that;

i) Reads the number N and the initial approximation $x_{0,}$

iii) Computes and prints the root to three decimal places and the number of iteration n.

b) Perform the dry for the flow chart taking $N = 54$.

(Answers; $\mathbf{1.835}$)

78. a) Show that the iterative formula for finding the reciprocal of a number N, is given by $x_{n+1} = x_n(2 - x_n N), n = 0, 1, 2, \ldots$

c) Draw a flow chart that;

ii) Reads the number N and the initial approximation $x_{0,}$

iv) Computes and prints the root to three decimal places and the number of iteration n.

e) Perform the dry for the flow chart taking $N = 54$ and $x_{0,} = 2.0$

(Answers; 1.768, n =4)

79. a) Show that the iterative formula for finding the reciprocal of a number N, is given by $x_{n+1} = x_n(2 - x_n N), n = 0, 1, 2, \ldots$

d) Draw a flow chart that;

iii) Reads the number N and the initial approximation $x_{0,}$

v) Computes and prints the root to three decimal places and the number of iteration n.

f) Perform the dry for the flow chart taking $N = 54$ $x_{0,} = 2.0$

(Answers; $\mathbf{0.019}$, $\mathbf{n = 6}$)

80. a) Show that the iterative formula for finding the reciprocal of a number N, is given by $x_{n+1} = x_n(2 - x_n N), n = 0, 1, 2, \ldots$

e) Draw a flow chart that;

iv) Reads the number N and the initial approximation $x_{0,}$

vi) Computes and prints the root to three decimal places and the number of iteration n.

g) Perform the dry for the flow chart taking $N = 54$ $x_{0,} = 2.0$

(Answers; 0.045, n = 4)

81. Show that the Newton Rapson formula for finding the arc sine of a number N is $x_{n+1} = x_n - \tan x_n + N\sec x_{n,}\ \ n = 0,1,2, \ldots$ Hence, find $\sin^{-1}(\frac{\pi}{5})$. Taking $x_0 = 0.5$ Giving your answer correct to three decimal places.

(Answers; $\mathbf{0.679}$)

82. a) Show that the iterative formula for finding the fourth root of a number N, is given by $x_{n+1} = \frac{3}{4}\left(x_n + \frac{N}{3x_n^3}\right), n = 0, 1, 2, \ldots$

d) Draw a flow chart that;

iii) Reads the number N and the initial approximation $x_{0,}$

iv) Computes and prints N and the fourth root to three decimal places and after 3 iteration.

e) Perform the dry for the flow chart taking $N = 40$ and $x_{0,} = 2.4$

(Answers; $\mathbf{2.515}$)

83. a) Show that the iterative formula for finding the fifth root of a number N, is given by $x_{n+1} = \frac{4}{5}\left(x_n + \frac{N}{4x_n^4}\right), n = 0, 1, 2, \ldots$

c) Draw a flow chart that;

- Reads the number N and the initial approximation $x_{0,}$
- Computes and prints the root to three decimal places with three iteration.

h) Perform the dry for the flow chart taking $N = 50$ and $x_{0,} = 2.0$

(Answers; $\mathbf{2.187}$)

84. Show that the iterative formula for finding the tenth root of a number N, is given by $x_{n+1} = \frac{9}{10}\left(x_n + \frac{N}{9x_n^9}\right), n = 0, 1, 2, \ldots$

d) Draw a flow chart that;

iii) Reads the number N and the initial approximation $x_{0,}$

iv) Computes and prints the root to three decimal places and 3 iterations.

i) Perform the dry for the flow chart taking $N = 44$.

(Answers; $\mathbf{1.460}$)

85.) Show that the iterative formula for finding the seventh root of a number N, is given by $x_{n+1} = \frac{6}{7}\left(x_n + \frac{N}{6x_n^6}\right), n = 0, 1, 2, \ldots$

d) Draw a flow chart that;

iii) Reads the number N and the initial approximation $x_{0,}$

iv) Computes and prints the root to three decimal places and with a maximum of three iterations.

e) Perform the dry for the flow chart taking $N = 54$.

(Answers; $\mathbf{1.768}$)

86. a) Show that the iterative formula for finding the seventh root of a number N, is given by $x_{n+1} = \frac{7}{8}\left(x_n + \frac{N}{7x_n^7}\right), n = 0, 1, 2, \ldots$

f) Draw a flow chart that;

vii) Reads the number N and the initial approximation $x_{0,}$

viii) Computes and prints the root to three decimal places and after 3 iterations.

f) Perform the dry for the flow chart taking $N = 70$.

(Answers; $\mathbf{1.835}$)

87. a) Show that the iterative formula for finding the reciprocal of a number N, is given by $x_{n+1} = x_n(2 - x_n N), n = 0, 1, 2, \ldots$

a) Draw a flow chart that;

g) Reads the number N and the initial approximation $x_{0,}$

iii) Computes and prints the root to three decimal places and after 3 iterations.

b) Perform the dry for the flow chart taking $N = 54\ x_{0,} = 2.0$

(Answers; $\mathbf{0.019}$)

88. (a) show that the newton Raphson's iterative formula for finding the natural logarithm of a number N is given by $x_{n+1} = \frac{e^{x_n}(x_n-1)+N}{e^{x_n}}$ for $n = 0, 1, 2, 3$ ....

(c) Draw a flow chart that reads the initial approximation x_0 and N, computes and prints the root after 4 iterations and gives the root to three decimal places.

(d) Perform the dry run for the flow chart taking $N = 10$ and $x_{0,} = 2$

(Answers; $\mathbf{2.303}$)

89. (a) show that the newton Raphson's iterative formula for finding the natural logarithm of a number N is given by $x_{n+1} = \frac{e^{x_n}(x_n-1)+N}{e^{x_n}}$ for $n = 0, 1, 2, 3$ ....

(e) Draw a flow chart that reads the initial approximation x_0 and N, computes and prints the root after 4 iterations and gives the root to three decimal places.

(f) Perform the dry run for the flow chart taking $N = 20$

(Answers; $\mathbf{2.996}$)

90. (a) show that the newton Raphson's iterative formula for finding the natural logarithm of a number N is given by $x_{n+1} = \frac{e^{x_n}(x_n-1)+N}{e^{x_n}}$ for $n = 0, 1, 2, 3$ ....

(g) Draw a flow chart that reads the initial approximation x_0 and N, computes and prints the root after 4 iterations and gives the root to three decimal places.

(h) Perform the dry run for the flow chart to estimate ln22

(Answers; $\mathbf{3.091}$)

82. (a) Derive the simplest formula based on Newton Raphson's method to show that for the equation 3x=In 3 it satisfies $x_{r+1} = \frac{1}{3}\left\{\frac{e^{3x}(3x_r-1)+3}{e^{3x}}\right\}$

(b) Draw a flow chat that:

i. Reads the initial approximate x_0,
ii. Accepts estimations to 4 significant figures.
iii. Prints the root to 4sf.

83. (a) Derive the simplest formula based on Newton Raphson's method to show that for the equation 3x=In3, it satisfies $x_{r+1} = \frac{1}{3}\left\{\frac{e^{3x}(3x_r-1)+3}{e^{3x}}\right\}$

(b) Draw a flow chat that:

iv. Reads the initial approximate x_0,

v. Accepts estimations to 4 significant figures and after 3 iterations.

vi. Prints the root to 4sf.

91. a) Show that the iterative formula for finding the eighth root of a number N, is given by $x_{n+1} = \frac{1}{8}\left(4x_n + \frac{N}{x_n^7}\right)$, $n = 0, 1, 2, \ldots$ hence, find $80^{1/2}$.

d) Draw a flow chart that reads the number N and the initial approximation $x_{0,}$ computes and prints the root to three decimal places and after 3 iteration.

(Answers; $\mathbf{8.944}$)

92. i) Show that the root of the equation $2x - 3\cos(x/2) = 0$ lies between 1 and 2.

iv) Use Newton Raphson's method to find the root of the equation in (i) above. Put your answer correct to three decimal places.

(Answers; $\mathbf{1.227}$)

a) Show that the equation $e^x - 4\sin x = 0$ has two roots between $x = 0$ and $x = 1.6$. (Answers; $\mathbf{0.371}$, $\mathbf{1.365}$)

a) By using the Newton Raphson's method to find the largest root of the equation $e^x - 4\sin x = 0$ correct to 3 decimal places.

(Answers; $\mathbf{1.365}$)

93. a) Show that the equation $e^x - 4\sin x = 0$ has two roots between $x = 0$ and $x = 1.6$.

a) By using the Newton Raphson's method to find the smaller root of the equation $e^x - 4\sin x = 0$ correct to 3 decimal places.

(Answers; $\mathbf{0.371}$)

94. a)Show that the equation $3\tan x + \frac{x}{3} = 0$ has a root between $x = 2$ and $x = 3.2$

c) Show that the Newton Raphson's method for finding the root of the equation $3\tan x + \frac{x}{3} = 0$ is $\frac{6x_n - 3\sin 2x_n}{6 + 2\cos 2x_n}$. Hence, approximate the root to three decimal places using the answer in (a) above as the initial approximation.

(Answers; $\mathbf{2.836}$)

95. Show graphically that the equation $x + \log_e x = 0.7$ has a positive root. Hence, use Newton Raphson's method to approximate the root to three decimal places.

(Answers; $\mathbf{0.856}$)

96. Show that the root of the equation $f(x) = e^x + x^3 - 4x = 0$ lies between 1 and 2. By using the N – R – M, find the root to two decimal places.

(Answers; $\mathbf{1.12}$)

97. Show that the root of the equation $f(x) = e^x + x - 4 = 0$ lies between 0.5 and 2. By using the N – R – M, find the root to three decimal places. Hence, illustrate your answer on a flow chart.

(Answers; $\mathbf{1.074}$)

98. Use graphical method to show that the equation $f(x) = e^x + x - 5 = 0$ has only one positive real root. Hence, using the N – R – M, find the root to three decimal places. Illustrate your answer on a flow chart.

(Answers; $\mathbf{1.307}$)

99. Use graphical method to show that the equation $f(x) = e^x - x - 10 = 0$ has only one positive real root. Hence, using the N – R – M, find the root to three significant figures. Illustrate your answer on a flow chart.

(Answers; $\mathbf{2.528}$)

100. a) Show that the equation $x^2 = 3x + 3$ has only two roots.

c) Use Newton Rapson Method the find the negative root correct to three decimal places.

d) Use linear interpolation to approximate the positive root to two decimal places.

(Answers; $\mathbf{b)}\ -\mathbf{0.791}$ $\quad \mathbf{c)}\ \mathbf{3.79}$)

101. a) Show that the root of the equation $x^2 = 3x - 1$ lies between 2 and 3. Hence, use linear interpolation to find the root to two decimal places.

(Answers; $\mathbf{2.62}$)

102. Show that the root of the equation $x^3 + 3x - 9 = 0$ lies between $x = 1$ and $x = 2$. Hence, use linear interpolation to find the root to two decimal places.

(Answers; $\mathbf{1.85}$)

103. a) Show that $x^3 - 3x^2 + 1 = 0$ has a real root between $x = 2$ and $x = 3$.

b) Using linear interpolation, find the first approximation for the root

c) Using the newton – Rapson formula, find the value of the root correct to 4 significant figures.

(Answers; $\mathbf{2.879}$)

104. Given the iterative method $x_{n+1} = \frac{1}{2}\left(x_n + \frac{10}{x_n}\right), n = 0, 1, 2, \ldots$

a) State the purpose of the iterative formula.

b) Show that the iterative formula is convergent for $x_0 = 2.5$, hence find the root to three decimal places.

(Answers; $\boldsymbol{a)}$ $\boldsymbol{To\ compute\ and\ print\ the\ squre\ root\ of\ 10.}$ $\boldsymbol{b)}$ $\mathbf{3.162}$)

105. Given the iterative method $x_{n+1} = \frac{1}{2}\left(x_n + \frac{15}{x_n}\right), n = 0, 1, 2, \ldots$

a) State the purpose of the iterative formula.
b) Show that the iterative formula is convergent for $x_0 = 3.6$, hence find the root to three decimal places.

(Answers; $\boldsymbol{a)}$ $\boldsymbol{To\ compute\ and\ print\ the\ squre\ root\ of\ 15.\ b)\quad 3.873}$**)**

106. Given the iterative method $x_{n+1} = \frac{1}{2}\left(x_n + \frac{45}{x_n}\right), n = 0, 1, 2, \ldots$

c) State the purpose of the iterative formula.
d) Show that the iterative formula is convergent for $x_0 = 6.7$, hence find the root to three decimal places.

(Answers; $\boldsymbol{a)}$ $\boldsymbol{To\ compute\ and\ print\ the\ squre\ root\ of\ 10.\ b)\quad 6.708}$ **)**

107. Given the two iterative formulae

$$x_{n+1} = \left(\frac{x_n^2-10}{2x_n-8}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(\text{A})$$

$$x_{n+1} = \left(8 - \frac{10}{x_n}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(\text{B})$$

a) Taking $x_0 = 1.3$, use each iterative formula thrice,
b) With respect to the answers in a) above, identify the most appropriate iterative formula. Hence, find the better approximation of the root to three decimal places.

(Answers; . $\boldsymbol{b)\ \ Formula\ A,\ \ 1.551}$**)**

108. Given the iterative method $x_{n+1} = \frac{1}{3}(x_n^3 + 1), n = 0, 1, 2, \ldots$ State the purpose of the iterative formula. Hence, taking $x_0 = 0.3$ find the root to three decimal places.

(Answers; $\boldsymbol{To\ compute\ and\ print\ the\ squre\ root\ of\ \ x^3 - 3x + 1 = 0,\ \ 1.532}$**)**

109. Show that the iterative formula for approximating the root of the equation

$x^3 - 3x_n - 3 = 0$ is $x_{n+1} = \left(\sqrt[3]{(3x_n + 3)}\right)$. Hence, using 2.1 as the initial approximation, find the root to 3 dps.

(Answers; $\mathbf{2.104}$)

110. Show that the iterative formula for approximating the root of the equation

$x^2 = 18$ is $x_{n+1} = \frac{1}{2}\left(x_n + \frac{18}{x_n}\right)$. Hence, using 4.2 as the initial approximation, find the root to 3 dps.

(Answers; $\mathbf{4.243}$)

111. Refer to UNEB **2001 No. 11**

112. **Refer to UNEB 2012 No. 11**

113. (a) Use the trapezium rule with 6 ordinates to approximate the value of $\int_0^2 \sqrt{(2 + \sin x)}\, dx$ correct to 3 decimal places.

114. (a) Use the trapezium rule with 5 sub intervals to estimate the value $\int_1^2 \frac{4}{x+2} dx$ of correct to 2 decimal places. Hence, determine the percentage error in approximating the function.

(b) Suggest how the percentage error can be reduced.

115. Estimate the value of $\int_0^1 \frac{dx}{1+x^3}$ by trapezium rule using five interval. Give your answer correct to 3 decimal places.

116. If A the area under the curve $y = \frac{X^2}{(2X^3+3)}$ between the x-axis x =2 and X = 3;

(a) Estimate the valve of a using trapezium rule with five strips. Give your answer to three significant figures.

(b) Find the exact valve of A hence;

(i) Determine the error in the estimation.

(ii) State how this error can be reduced.

117. If W the area under the curve $y = \frac{4X^3}{(X^4+15)}$ between the x-axis x =1 and X = 2.2;

(a) Estimate the valve of a using trapezium rule with six sub - ordinates. Give your answer to three decimal places.

(b) Find the exact valve of W hence;

[i]. Determine the percentage error in the estimation.

[ii]. State how this error can be reduced.

118. **(i)** Use the trapezium rule with 6 sub intervals to estimate the value $y = 4^x$ between the x-axis,$x = 1$ and $x = 1.5$ correct to 4 significant figures. Hence, determine the relative error in approximating y.

(iii) Find the exact error of $\int_1^{1.5} 4^x \, dx$

(iv) Find the percentage error in calculations (i) and (ii) above.

119. a) Use the trapezium rule with 4 ordinates to estimate the value $p = 5^x$ between the x-axis,$x = 1$ and $x = 2$ correct to 3 decimal places. Hence, determine the relative error in approximating p.

(v) Find the exact error of $\int_1^2 5^x \, dx$

(vi) Find the percentage error in calculations (i) and (ii) above.

120. (a) Draw a follow chart that can compute and print the sum cubes of the first six counting numbers.

(b)The taxation on the income of employees of a certain company is given in by,

Taxable income, I (shs)	Taxation rate (%)
0 – 100,000	0
100,001 - 200,000	8
200,001 - 300,000	15
300,001 and above	20

Construct a flow chart that reads the taxable income I, calculates and prints the tax amount T and the taxable income. Hence perform a dry run for employees earning; sh.154,000, sh. 250,000 and sh. 558,000 respectively.

(Answers; $\boldsymbol{a}$) $\mathbf{441}$. $\boldsymbol{b}$) $\mathbf{12,320}$, $\mathbf{37,500}$, $\mathbf{111,600}$)

115. (a) Draw a follow chart that can compute and print the sum of the first ten odd numbers.

(b)The taxation on the income of employees of a certain company is given in by,

Taxable income, i (shs)	Taxation rate T(%)
80000 – 100,000	2
100,001 - 300,000	12
300,001 - 600,000	20
600,001 and above	31

Construct a flow chart that reads the taxable income i, calculates and prints the tax amount T and the taxable income. Hence perform a dry run for employees earning; sh.154,000, sh. 400,000 and sh.1,000,000 respectively.

(Answers; $\boldsymbol{a}$) $\mathbf{100}$. $\boldsymbol{b}$) $\mathbf{18480}$, $\mathbf{80,000}$, $\mathbf{310,000}$)

116. (a) Draw a follow chart that can compute and print the factorial of a number N for which N in a counting number including zero up to 7.

(b)The taxation on the income of employees of a certain company is given in by,

Taxable income, I (shs)	Taxation rate (%)
30000– 100,000	2
100,001 - 200,000	8
200,001 - 300,000	15
800,001 and above	20

Construct a flow chart that reads the taxable income I, calculates and prints the tax amount T and the taxable income. Hence perform a dry run for employees earning; sh.40,000, sh. 300,000 and sh. 3,000,000 respectively.

(Answers; $\boldsymbol{a}$) $\mathbf{5040}$. $\boldsymbol{b}$) $\mathbf{800}$, $\mathbf{45,000}$, $\mathbf{600,000}$)

117. below the table gives t and the corresponding values of $g(t)$

t	15	30	45	60
$g(t)$	1.8148	3.1624	5.0946	7.3456

Use linear interpolation and extrapolation to solve for;

(i) $y = 2.5t \ when \ g(t) = 6.382$

(ii) $g(39.5)$

(iii) $t \ when \ g(t) = 2.992.$

118. The table below shows the variation between x and y

x	1.00	1.72	2.00	2.37
y	2.718	9.605	14.778	25.353

Use linear interpolation and extrapolation to solve for;

(i) The value of x when y=5.678

(ii) y when x = 2.55

(iii)x when y = 12.002

119. The table below represents the variation between x and y

x	-1.62	-1.51	-1.02	1.00
y	37.34	33.45	20.49	2.72

Use linear interpolation and extrapolation to solve for;

(i) $x \text{ when } y = 11.98$

(ii) $y \text{ when } x = 0$

(iii)$y \text{ when } x = -2.05$

120. The table below shows the time and the corresponding velocity for a particle projected vertically upwards with other factors affecting its motion.

Time in minutes	2.66	2.87	3.01	3.55
Velocity in meters per second	2.794	1.633	0.810	-2.733

Using linear interpolation and extrapolation find;

(i) The time taken for the particle to reach the maximum height

(ii) Initial velocity of projection

(iii)Velocity of the particle when time is 174seconds after projections.

121. The table below indicates the variation between $h \text{ and } t$.

h	1.5	2.0	2.5	3.0
t	0.5	0.8	1.1	1.5

Find the value of;

(i) $h \text{ when } t = -1.0$

(ii) $t \text{ when } h = 2.38$

(iii)$t \text{ when } h = 3.5$

122. The table is an extract from table of sine's.

$10.\Phi$	0^1	6^1	12^1	18^1	24^1	30^1
$\sin 10.\Phi°$	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822

Determine;

(i) Sin $10^0 16^1$

(ii) $Sin^{-1} 0.1747$

123. The table below is an extract from the table of cosine of an angle X^0.

X^0	50.0^0	50.2^0	50.40^0	50.6^0
Cos X^0	0.6428	0.6401	0.6374	0.6347

Using linear interpolation, find

(i) Cos 50.3^0

(ii) $Cos^{-1} 0.6361$

124. Given that $(2.65)^3 = 17.576$, $(2.7)^3 = 19.638$. Using linear interpolation or extrapolation, estimate the value of:

(i) $(2.65)^3$

(ii) X such that $x^3 = 23.25$.

125. The data below is of exponential function e^x, where $e^{0.30} = 1.3499$ and $e^{0.35} = 1.4191$, estimate using linear interpolation the following;

(i) $e^{0.32}$

(iii) x for which $e^x = 1.39$

126. The table below shows the cost y shillings for fuel consumed by a vehicle in a distance x kilometers.

Distance(x km)	10	20	30	40
Cost(Sh. y)	14800	15600	16400	17200

Use linear interpolation or extrapolation to calculate the;

(a) Cost of fuel consumed for a distance of 45km.

(b) Distance Kabito travelled if he put fuel of shs. 16000.

127. (a) The table below shows the cost y shillings for hiring a motor cycle for a distance x kilometer.

Distance (x km)	10	20	30	40
Cost (shs)	2800	3600	4400	5200

Use linear interpolation or extrapolation to calculate the;

(i) Cost of hiring the motorcycle for a distance of 45 km

(ii) Distance Mukasa travelled if he paid $shs\ 4000$.

(b). (i) The public mean cost from Lukaya to Kampala city is $shillings 12000$ for a total distance of 120 km.

(ii) If the distance from Kyengera town to Kampala is $20\ km$, how much should be paid from Lukaya to Kyengera.

(iii) Peter was driving from Lukaya to Kampala and his car got a puncture after $60\ km$, how much is he supposed to pay for the remaining distance to Kampala.

128. If x_1=3.622 and x_2 =1.95, determine for the range within which the exact values of

(i) $x_1 x_2(x_1 - x_2)$

(ii) $\frac{x_2 - x_1}{x_1 x_2}$

129. (a) Given y=$\frac{\cos q}{x}$, show that the percentage error in y is $\left[\left|\frac{\Delta x}{x}\right| + |\tan q||\Delta q|\right] \times 100\%$ whereΔx and Δq are errors in x and q respectively.

(b) If $x = 6.4, q = 120°$, and the percentage errors in x and q are 4 and 9 respectively, determine the percentage error in calculating y.

f) (a) Given the following values; $x = 7.6, y = 45, z = 0.35$, recorded to the given number of decimal places, find the percentage error in the expression $\frac{z}{x+y}$

(b) Given that $x = 3.57, y = -4.291$ and $z = 6.7955$ are rounded off to the given decimal places indicated,

i. Find the maximum possible errors in x, y and z

ii. Find the limits within which the exact value of the expression $\frac{x}{y-z}$ lies.

129. the of a right cylinder is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h, the height of the cylinder. Derive an expression for the maximum error made in the volume when the radius and the height change by Δr and Δh respectively.

b) The numbers $A = 12.4$, $B = 29.444$ and $C = 2.25$ are each rounded off with percentage errors

2.5%, 0.05% and 1% respectively. Find the;

(i) limits within which the exact value of $\frac{\text{A}}{(\text{B}-\text{C})^3}$ lies'

(ii) Percentage error made in $\frac{\text{A}}{(\text{B}-\text{C})^3}$

Given that the numbers p and q are rounded off with errors ep and eq. Show that the maximum relative error

130. The table below shows the values of the function $f(x)$

x	1.8	2.0	2.4	2.4
$f(x)$	0.532	0.484	0.436	0.384

Use linear interpolation to find;

h) $f(\,2.08)$,

ii) x if $f(x) = 0.5$,

iii) $f^{-1}(\,0.666)$.

131. The table below shows an extract from the table of $cosx$

$80.\theta$	0'	10'	20'	30'	40'	50'
$cos80.\theta$	0.1736	0.1708	0.1679	0.1650	0.1622	0.1593

Determine; i) $\cos 80^0\ 36'$

a) $\cos^{-1} 0.1685$

b) $\cos^{-1} 0.1777$

c) $\cos 80^0\ \ 70'$

d) $\cos\ 80.1333$

(Answers; $\boldsymbol{a)}\ \mathbf{80^0\ 17.931'}.\ \boldsymbol{b)}\ \ \mathbf{79.7560^0},\ \ \boldsymbol{c)}\ \mathbf{0.1535}\ \ \boldsymbol{d)}\ \mathbf{0.1714}$)